Day 6

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 2 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 3 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \quad \vec{v}_3 = \begin{bmatrix} 0 \\ 5 \\ 9 \\ 5 \end{bmatrix}$$

The RREF of $A = \begin{bmatrix} 1 & 3 & 0 \\ 2 & 1 & 5 \\ 3 & 0 & 9 \\ 2 & 1 & 5 \end{bmatrix}$

is $R = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

1. Are $\vec{v}_1, \vec{v}_2, \vec{v}_3$ linearly dependent, or linearly independent?

2. $\vec{v}_4 = \begin{bmatrix} -16 \\ 3 \\ 15 \\ 3 \end{bmatrix}$ and the RREF...

...of $\begin{bmatrix} 1 & 3 & 0 & -16 \\ 2 & 1 & 5 & 3 \\ 3 & 0 & 9 & 15 \\ 2 & 1 & 5 & 3 \end{bmatrix}$ is $\begin{bmatrix} 1 & 0 & 3 & 5 \\ 0 & 1 & -1 & -7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

Use this information to find $x, y, z \in \mathbb{C}$ such that $\vec{v}_4 = x\vec{v}_1 + y\vec{v}_2 + z\vec{v}_3$.

(3) If $N(B) = \left\langle \left\{ \begin{bmatrix} 0 \\ 5 \\ 1 \end{bmatrix} \right\} \right\rangle$, then the columns of B are linearly ________.

(4) If the RREF of matrix C has 8 rows, 5 columns, and 5 pivots, then the columns of C are linearly ________.