

① Give an example of 2×2 invertible matrices A, B such that $A + B$ is not invertible and $AB \neq A^{-1}B^{-1}$.

② Give an example of a 2×2 matrix A such that $A^{-1} = A^* \neq I_2$.

③ There is a matrix $E = \begin{bmatrix} 1 & ? \\ ? & 1 \end{bmatrix}$ such that $E \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ adds five copies of $\begin{bmatrix} a & b \end{bmatrix}$ to $\begin{bmatrix} c & d \end{bmatrix}$ to make $\begin{bmatrix} a & b \\ 5a+c & 5b+d \end{bmatrix}$. Find E and then find E^{-1} .

④ If $A \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$ and $A \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$, is A invertible? Yes, no, or not enough information?