(1) Which of the following are subspaces of $\mathbb{C}^3$?

Day 14

$P = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \middle| \; x+y+z=0 \right\}$, $Q = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \middle| \; x+y=0 \right\}$,

$R = R\left(\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \right)$, $S = C\left(\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \right)$,

$T = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \middle| \; x+y+z=1 \right\}$, $U = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \middle| \; xy=0 \right\}$,

$V = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \middle| \; x+y=0 \text{ or } y+z=0 \right\}$, $W = N\left(\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \right)$,

$X = L\left(\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \right)$, $Y = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \middle| \; x+y=0 \text{ and } y+z=0 \right\}$.

(2) Which of the following are subspaces of P_5? (Notation: $p' = dp/dx$; $p(a) = (p \text{ at } x=a)$) (14

$A = \langle \{5-x, x^2, x^5\} \rangle$, $B = \{p \in P_5 \mid p(3) = 0\}$,

$C = \{p \in P_5 \mid p' = 0\}$, $D = \{p \in P_5 \mid p'''(3) = 0\}$,

$E = \{p \in P_5 \mid 2p(1) = p'(0) + p''(-1)\}$,

$F = \{p \in P_5 \mid \int_0^5 p(t)\,dt = \int_1^2 p(t)\,dt\}$,

$G = \{p \in P_5 \mid \int_0^5 p(t)\,dt = 3\}$, $H = P_4$,

$I = \{p \in P_5 \mid \frac{d}{dx}(x^2 p(x)) = 3x\,p(x)\}$,

$J = \{p \in P_5 \mid \frac{d}{dx}(x^2 p(x)) = 3x\}$, $K = \{0\}$.

(3) Which of the following are subspaces of M_{33}?

$U = \{A \in M_{33} \mid A \text{ is unitary}\}$,

$H = \{A \in M_{33} \mid A \text{ is self-adjoint}\}$,

$N = \{A \in M_{33} \mid A\begin{bmatrix}1\\2\\3\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}\}$,

$E = \{A \in M_{33} \mid A\begin{bmatrix}1\\2\\3\end{bmatrix} = A\begin{bmatrix}4\\5\\6\end{bmatrix}\}$,

$R = \{A \in M_{33} \mid \begin{bmatrix}7&3&5\\8&2&1\end{bmatrix}A = \begin{bmatrix}0&0&0\\0&0&0\end{bmatrix}\}$,

$V = \{A \in M_{33} \mid A \text{ is invertible}\}$,

$S = \{A \in M_{33} \mid A \text{ is singular}\}$,

$T = \{A \in M_{33} \mid A = A^t\}$.