Day 19

(1) Find $\det\left(\begin{bmatrix} 1 & 0 & 0 & 0 \\ 5 & 2 & 0 & 0 \\ 6 & 7 & 3 & 0 \\ 8 & 9 & 10 & 4 \end{bmatrix}\right)$ & $\det\left(\begin{bmatrix} -1 & 4 & -i & 9 \\ 0 & i & 7 & 0 \\ 0 & 0 & 2 & -6 \\ 0 & 0 & 0 & 3 \end{bmatrix}\right)$.

These are easier than they might look.

(2) Suppose $B = \begin{bmatrix} 1 & 5 & 3 \\ 0 & 4 & 0 \\ 0 & 0 & 7 \end{bmatrix} = E_3 E_2 E_1 A$ where

E_1 performs row operation $-R_2 + R_3$,
E_2 performs row operation $R_1 \leftrightarrow R_2$,
and E_3 performs $\frac{1}{2} R_1$.

What is $\det(A)$?

(3) Give examples of 2×2 matrices A & B such that $\det(A+B) \neq \det(A) + \det(B)$.

(4) Suppose A is 5×5 and $\det(A) = 3$.
What is $\det(2A)$? $\det(A^2)$? $\det(A^{-1})$?

(5) What is $\det\left(\begin{bmatrix} \frac{1}{5} & \frac{3}{7} \end{bmatrix}\right)$?