Day 22

① Which of these matrices are diagonalizable?

$A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$,

$C = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 1 \\ 0 & 1 & 3 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 4 & 5 \\ 0 & 2 & 6 \\ 0 & 0 & 3 \end{bmatrix}$, $E = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$

---

② Given $A = \begin{bmatrix} 2 & 9 \\ 9 & 2 \end{bmatrix}$, find $\sqrt{A}$, A^{100}, e^A and e^{tA}.

③ If $dx/dt = 2x + 9y$ and $dy/dt = 9x + 2y$ and $x(0) = 0$ and $y(0) = -3$, find $x(t)$ & $y(t)$.

~~④ If A is diagonalizable, 5×5, $A^5 = A$, $A^4 \neq I$, and A is real~~

Day 22

(4) Give two examples of a real 4×4 matrix with eigenvalues $3i$ & $-3i$ and no others. Make the first diagonalizable and the second not.

---

(5)

$$1+1+1 = 3 = t_1$$

$$1+1+3 = 5 = t_2$$

$$1+3+5 = 9 = t_3$$

$$3+5+9 = 17 = t_4$$

$$5+9+17 = 31 = t_5$$

and so on...

$t_n = \alpha_1 \lambda_1^n + \alpha_2 \lambda_2^n + \alpha_3 \lambda_3^n$ for all $n = 1, 2, 3, \ldots$ if you get the six constants $\alpha_1, \alpha_2, \alpha_3, \lambda_1, \lambda_2, \lambda_3$ right. Find a polynomial $p(\lambda)$ whose roots are λ_1, λ_2, and λ_3.