

① Let  $F: P_3 \rightarrow P_3$  where

$$(Fp)(x) = (x^2 - 1)p''(x) + 7xp'(x+5) - p'(3) + p'''(8x^2) - (x^2 p(x))'' + \int_{x^2}^2 p'''(x+1)dx.$$

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Is  $F$  linear?

② Let  $G: M_{25} \rightarrow M_{52}$ ,  $GA = A^t$ .

Is  $G$  linear?

③ Let  $H: M_{44} \rightarrow M_{44}$ ,  $HA = A^* + A$ .

Is  $H$  linear?

④ Let  $J: M_{33} \rightarrow M_{33}$ ,  $JA = 3A + I$ .

Is  $J$  linear?

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⑤ Let  $K: P_2 \rightarrow P_4$  where

$(Kp)(x) = (p(x))^2$ . Is  $K$  linear?

⑥ Find the matrix of  $T: U \rightarrow L$

where  $U = \{A \in M_{33} \mid A \text{ is upper triangular}\}$

&  $L = \{A \in M_{33} \mid A \text{ is lower triangular}\}$

&  $TA = A^t$ , after finding bases

of  $U$  &  $L$ . (The matrix of  $T$  depends

on which bases you choose.)

⑦ Let  $Q_1: M_{22} \rightarrow M_{22}$  where  $Q_1 A$  is

$A$  after ~~adding~~ adding column 2 to column 1

For basis  $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ ,

Find the matrix of  $Q_1$ .

(8) Let  $Q_2: M_{22} \rightarrow M_{22}$  ~~where~~ where  $Q_2 A$  is  $A$  after adding row 2 to row 1. Using the basis from (7), find the ~~matrix~~ matrix of  $Q_2$ .

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(9) Give an example  $A \in M_{22}$  where  $Q_1 Q_2 A \neq Q_2 Q_1 A$ .

(10) Define  $Q_1 Q_2 - Q_2 Q_1: M_{22} \rightarrow M_{22}$  by

$$(Q_1 Q_2 - Q_2 Q_1) A = Q_1 Q_2 A - Q_2 Q_1 A.$$

Find a basis for  $\{A \in M_{22} \mid Q_1 Q_2 A = Q_2 Q_1 A\}$

by first finding a basis for the null space

of the matrix of  $Q_1 Q_2 - Q_2 Q_1$ ,

with respect to the basis from (7).