① Let $F: P_3 \to P_3$ where

$$(Fp)(x) = (x^2-1)p''(x) + 7xp'(x+5) - p'(3)$$
$$+ p'''(8x^2) - (x^2 p(x))'' + \int_{x^2}^{2} p'''(x+1)dx.$$

Is F linear?

② Let $G: M_{25} \to M_{52}$, $GA = A^t$.

Is G linear?

③ Let $H: M_{44} \to M_{44}$, $HA = A^* + A$.

Is H linear?

④ Let $J: M_{33} \to M_{33}$, $JA = 3A + I$.

Is J linear?

(5) Let $K: P_2 \rightarrow P_4$ where

$(Kp)(x) = (p(x))^2$. Is K linear?

(6) Find the matrix of $T: U \rightarrow L$

where $U = \{ A \in M_{33} \mid A$ is upper triangular$\}$

& $L = \{ A \in M_{33} \mid A$ is lower triangular$\}$

& $TA = A^t$, _after_ finding bases

of U & L. (The matrix of T depends

on which bases you choose.)

(7) Let $Q_1: M_{22} \rightarrow M_{22}$ where $Q_1 A$ is

A after ~~adding~~ adding column 2 to column 1

For basis $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$,

Find the matrix of Q_1.

(8) Let $Q_2 : M_{22} \longrightarrow M_{22}$ ~~✗✗~~ where $Q_2 A$ is A after adding row 2 to row 1. Using the basis from (7), find the ~~✗✗✗~~ matrix of Q_2.

(9) Give an example $A \in M_{22}$ where $Q_1 Q_2 A \neq Q_2 Q_1 A$.

(10) Define $Q_1 Q_2 - Q_2 Q_1 : M_{22} \longrightarrow M_{22}$ by

$(Q_1 Q_2 - Q_2 Q_1) A = Q_1 Q_2 A - Q_2 Q_1 A.$

Find a basis for $\{A \in M_{22} \mid Q_1 Q_2 A = Q_2 Q_1 A\}$ by first finding a basis for the null space of the matrix of $Q_1 Q_2 - Q_2 Q_1$ with respect to the basis from (7).