Day 28

(1) Let $T: M_{22} \to M_{22}$ where $TA = A^t$. Give a basis of M_{22} that makes the matrix of T diagonal.

(2) $R: \mathbb{C}^3 \to \mathbb{C}^3$ with matrix $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ with respect to basis $\vec{e}_1, \vec{e}_2, \vec{e}_3$ is actually a 120° rotation. Find an orthonormal basis of $\mathbb{C}^3$ with respect to which the matrix of R is $\begin{bmatrix} \cos\frac{2\pi}{3} & -\sin\frac{2\pi}{3} & 0 \\ \sin\frac{2\pi}{3} & \cos\frac{2\pi}{3} & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

(3) Explain why there is no basis of P_3 that makes the matrix of $D: P_3 \to P_3$, where $(Dp)(x) = p'(x)$, diagonal.

Day 28

(4) Suppose $F: P_2 \to P_3$ has matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 2 \\ 1 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

with respect to [input basis $1, x+1, x^2+x+1$] & [output basis $(x-1)^3, (x-1)^2, x-1, 5$]. Find the matrix B of F with respect to input basis $1, x, x^2$ & output basis $1, x, x^2, x^3$.

(5) Which of the following matrices have orthonormal bases of eigenvectors?

$$\begin{bmatrix} 2 & 1 \\ 1 & 5 \end{bmatrix}, \begin{bmatrix} 2 & -1 \\ 1 & 5 \end{bmatrix}, \begin{bmatrix} 5 & -1 \\ 1 & 5 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & i \end{bmatrix},$$

$$\begin{bmatrix} 0 & -i \\ i & 1 \end{bmatrix}, \begin{bmatrix} 0 & i \\ i & 1 \end{bmatrix}$$