

① Let $T: M_{22} \rightarrow M_{22}$ where $TA = A^t$. Day
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 Give a basis of M_{22} that makes the matrix of T diagonal.

② $R: \mathbb{C}^3 \rightarrow \mathbb{C}^3$ with matrix $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$
 with respect to basis $\vec{e}_1, \vec{e}_2, \vec{e}_3$ is actually a 120° rotation. Find an orthonormal basis of \mathbb{C}^3 with respect to which the matrix of R is $\begin{bmatrix} \cos \frac{2\pi}{3} & -\sin \frac{2\pi}{3} & 0 \\ \sin \frac{2\pi}{3} & \cos \frac{2\pi}{3} & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

③ Explain why there is no basis of P_3 that makes the matrix of $D: P_3 \rightarrow P_3$, where $(Dp)(x) = p'(x)$, diagonal.

④ Suppose $F: P_2 \rightarrow P_3$ has matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 2 \\ 1 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \text{ with respect to } [\text{input basis}$$

$1, x+1, x^2+x+1]$ & [output basis ~~$1, x, x^2$~~

$(x-1)^3, (x-1)^2, x-1, 5]$. Find the matrix B

of F with respect to input basis $1, x, x^2$ & output basis $1, x, x^2, x^3$.

⑤ Which of the following matrices have orthonormal bases of eigenvectors?

$$\begin{bmatrix} 2 & 1 \\ 1 & 5 \end{bmatrix}, \begin{bmatrix} 2 & -1 \\ 1 & 5 \end{bmatrix}, \begin{bmatrix} 5 & -1 \\ 1 & 5 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & i \end{bmatrix},$$

$$\begin{bmatrix} 0 & -i \\ i & 1 \end{bmatrix}, \begin{bmatrix} 0 & i \\ i & i \end{bmatrix}$$

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