More exercises to try: #2 & 6] p. 21
#9, 12, 16, 17]

Quiz on p. 12–17 on Wednesday

Let's look at #4:

Let E nonempty; L ordered set, and $\underbrace{E \subseteq L}_{\text{same as } \subset}$. Suppose α a lower bound of E & $\boxed{\beta \text{ an upper bound of } E.}$

Prove that $\alpha \leq \beta$.

What happens if $\underbrace{E \text{ is empty}}_{E = \emptyset}$?

↓

Every α is a lower bound & every β is an upper bound, so $\alpha \nleq \beta$ is possible.

$\boxed{\forall x \in E \quad x \leq \beta}$

Pick $x \in E$. By definition,

$\forall y \in E \quad \alpha \leq y$ &

$\forall y \in E \quad y \leq \beta$.

So, $\alpha \leq x$ & $x \leq \beta$. So, $\alpha \leq \beta$. $\square$

$\underbrace{P \Rightarrow Q}$ "if P, then Q"

`$P\Rightarrow Q$` "P implies Q"

$P \Longleftrightarrow Q$ "P if and only if Q"

$P \Longleftarrow Q$ "P only if Q"

"if Q, then P"

$(P \Rightarrow Q) \Longleftrightarrow \underbrace{(\overset{\text{not}}{\neg} Q \Rightarrow \neg P)}_{\text{contrapositive}}$

$(\neg \forall x \in E \ P) \Longleftrightarrow (\exists x \in E \ \neg P)$

$(\neg \exists x \in E \ P) \Longleftrightarrow (\forall x \in E \ \neg P)$

`$\neg\exists x \in E P\Leftrightarrow\forall x \in E \neg P$`

$\neg(P \text{ and } Q) = (\neg P) \text{ or } (\neg Q)$
$\neg(P \text{ or } Q) = (\neg P) \text{ and } (\neg Q)$

"P or Q" means at least one
of P & Q is true

"P or Q"	$P \vee Q$ \vee	Use sparingly
"P and Q"	$P \wedge Q$ \wedge	

Prove $b^n - a^n = (b-a)(b^{n-1} + ab^{n-2}$
$+ a^2 b^{n-3}$
$+ a^3 b^{n-4}$
$\vdots$
$+ a^{n-4} b^3$
$+ a^{n-3} b^2$
$+ a^{n-2} b + a^{n-1})$

for all $n = 1, 2, 3, 4, 5, 6, \ldots$

Proof by induction:

Prove case $n=1$.

Then prove case $n=k$ implies case $n=k+1$, for all k.

Be more precise:

$$b^n - a^n = \left(\sum_{s=1}^{n} a^{s-1} b^{n-s}\right)(b-a)$$

Case $n=1$:

$$\left(\sum_{s=1}^{1} a^{s-1} b^{n-s}\right)(b-a) \rightarrow$$

$$\rightarrow \quad = \left(a^{1-1} b^{1-1}\right)(b-a) \rightarrow$$

$$\rightarrow \quad = \left(a^0 b^0\right)(b-a) = (1)(b-a)$$

$$\rightarrow \quad = b-a = b^1 - a^1 \quad \checkmark$$

Case $n=k$ implies case $n=k+1$:

Assume $\boxed{b^k - a^k} = \left(\sum_{s=1}^{k} a^{s-1} b^{k-s}\right)(b-a).$

Goal: $b^{k+1} - a^{k+1} = \left(\sum_{s=1}^{k+1} a^{s-1} b^{k+1-s}\right)(b-a)$

$$\left(\sum_{s=1}^{k+1} a^{s-1} b^{k+1-s}\right)(b-a)$$

$$= \left(\left(\sum_{s=1}^{k} a^{s-1} b^{k+1-s}\right) + \overbrace{a^{k+1-1} b^{k+1-(k+1)}}^{s=k+1}\right)(b-a)$$

$$= \left(\left(\sum_{s=1}^{k} a^{s-1} b^{k+1-s}\right) + a^k b^0\right)(b-a)$$

$$= \left(\left(\sum_{s=1}^{k} a^{s-1} b^{k-s}\right) b + a^k\right)(b-a)$$

$$= \boxed{\left(\sum_{s=1}^{k} a^{s-1} b^{k-s}\right)(b-a)}\, b + a^k(b-a)$$

$$= \boxed{(b^k - a^k)}\, b + a^k(b-a)$$

$$= b^{k+1} - a^k b + a^k b - a^{k+1}$$

$$= b^{k+1} - a^{k+1}. \quad \checkmark \quad \square$$