Example of upper & lower limits:

$$x_{3n} = 0; \quad x_{3n+1} = \frac{2+n}{1+n}; \quad x_{3n+2} = -\frac{2+n}{1+n}$$

n	x_n	$\sup\{x_m : m \geq n\}$	$\inf\{x_m : m \geq n\}$
0	0	2/1	−2/1
1	2/1	2/1	−2/1
2	−2/1	3/2	−2/1
3	0	3/2	−3/2
4	3/2	3/2	−3/2
5	−3/2	4/3	−3/2
6	0	4/3	−4/3
7	4/3	4/3	−4/3
8	−4/3	5/4	−4/3
⋮	⋮	↓ $\lim_{n\to\infty}$	↓ $\lim_{n\to\infty}$

$$\limsup_{n\to\infty} x_n = 1 \qquad -1 = \liminf_{n\to\infty} x_n$$