Quiz: Derivatives...

A) ... are always continuous

[✓ B) ... have no simple discontinuities]

C) ... have only simple discontinuities

Suggested exercizes

Ch. 5 # 8, 9, 12, 16

Correct statement of

Taylor's Thm:

If $f^{(n-1)}$ is cts on $[a,b]$

& $f^{(n)}$ exists on all of (a,b),

and $\alpha \neq \beta$ are in $[a,b]$, then,

for some x between α & β,

$$
\begin{aligned}
f(\beta) = f(\alpha) &+ f'(\alpha)(\beta-\alpha) \\
&+ f''(\alpha)(\beta-\alpha)^2/2 \\
&+ f^{(3)}(\alpha)(\beta-\alpha)^3/6 \\
&\vdots \\
&+ f^{(n-1)}(\alpha)(\beta-\alpha)^{n-1}/(n-1)! \\
&+ f^{(n)}(x)(\beta-\alpha)^n/n!
\end{aligned}
$$

Taylor series of $\ln x$ is good approximation for $x \in [\frac{1}{2}, 2]$. Estimating $\ln 5$: (Use precomputed $\ln 2$.)

$$\ln 5 = -\ln(5^{-1}) = -\ln \frac{1}{5} = -\ln\left(\frac{1}{4} \cdot \frac{4}{5}\right)$$

$$= -\left(\ln \frac{1}{4} + \ln \frac{4}{5}\right) = 2 \ln 2 - \ln \frac{4}{5}$$

For some $x \in [4/5, 1]$, we have

$$\ln \frac{4}{5} = \ln 1 + (\ln' 1)\left(\frac{4}{5} - 1\right)$$
$$+ (\ln'' 1)\left(\frac{4}{5} - 1\right)^2 / 2$$
$$+ (\ln^{(3)} x)\left(\frac{4}{5} - 1\right)^3 / 6$$

$$\ln' x = +x^{-1} \quad \ln'' x = -1 x^{-2}$$

$$\ln^{(3)} x = +2! \, x^{-3}, \quad \left(\ln^{(n)} x = (-1)^{n+1} (n-1)! \, x^{-n}\right)$$

$$\ln \frac{4}{5} = 0 + \overbrace{1(-1/5)}^{-1/5} \overbrace{- 1(-1/5)^2 / 2}^{-1/50} + \frac{\ln^{(3)} x}{6 \cdot 5^3}$$

$$\left| \frac{\ln^{(3)} x}{6 \cdot 5^3} \right| = \frac{2}{(5x)^3 \cdot 6} = \frac{1}{3(5x)^3} \leq \frac{1}{3 \cdot 4^3}$$

$$-\frac{11}{50} - \frac{1}{192} \leq \ln \frac{4}{5} \leq -\frac{11}{50} + \frac{1}{192}$$

$$2 \ln 2 + \frac{11}{50} - \frac{1}{192} \leq \ln 5 \leq 2 \ln 2 + \frac{11}{50} + \frac{1}{192}$$

L'Hospital's Rule:

If ① $f, g \to 0$ as $x \to a$,
② $f'/g' \to L$ as $x \to a$, and
③ $g' \neq 0$ near a,

then $f/g \to L$ as $x \to a$.

Warnings:

$$\lim_{x \to 0} \frac{\overset{\not\to 0}{\cos(x)}}{\underset{\searrow 0}{x}} \neq \lim_{x \to 0} \frac{-\sin(x)}{1} \overset{?}{=} 0$$

$$\lim_{x \to 0} \frac{x + x^2 \sin(1/x)}{x + x^2} = \lim_{x \to 0} \frac{1 + x\sin(1/x)}{1 + x}$$

$$= \frac{1+0}{1+0} = 1$$

$$\lim_{x \to 0} \frac{1 + 2x\sin(1/x) - \cos(1/x)}{1 + 2x}$$ does not equal 1; in fact, $\begin{cases} \limsup = 2 \\ \liminf = 0. \end{cases}$