$X = \mathbb{R}$ $\qquad E \subset X$

If $E = [0, 1)$, then

$\{ (-1, \frac{1}{2}), (-1, \frac{2}{3}), (-1, \frac{3}{4}), \ldots \}$

is an open cover of E in X

with no finite subcover.

Thus, $[0, 1)$ is not compact.

If $E = [0, \infty)$, then

$\{ (-1, 1), (-1, 2), (-1, 3), \ldots \}$

is an open cover of E in X

with no finite subcover.

Thus, $[0, \infty)$ is not compact.

$X = \mathbb{R}$ $\qquad$ $E \subset X$

~~If $E = [0, 1]$, then $\{(-1, \frac{1}{5})\} \cup \{(2, \frac{1}{2})\ (1, \frac{1}{4}), (\frac{1}{2}, \frac{1}{8}), (\frac{1}{4}, \frac{1}{16})$~~

If $E = [0, 1]$, then E is compact.

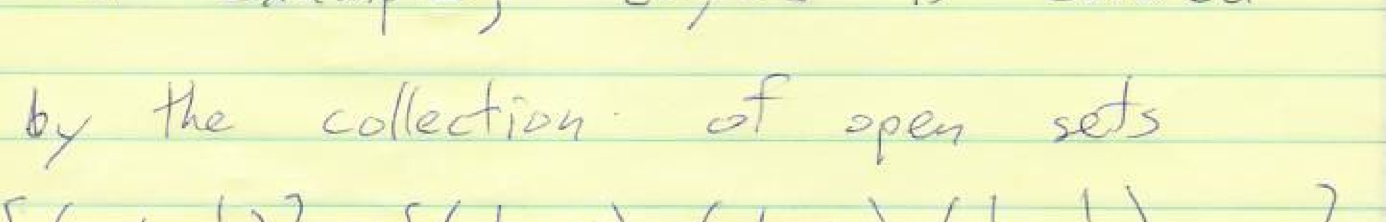
For example, $[0, 1]$ is covered by the collection of open sets

$$\{(-1, \tfrac{1}{5})\} \cup \{(\tfrac{1}{2}, 2), (\tfrac{1}{4}, 1), (\tfrac{1}{8}, \tfrac{1}{2}), \ldots\}$$

~~and~~ and this open cover has

$$\{(-1, \tfrac{1}{5}), (\tfrac{1}{8}, \tfrac{1}{2}), (\tfrac{1}{4}, 1), (\tfrac{1}{2}, 2)\}$$

as a finite subcover:



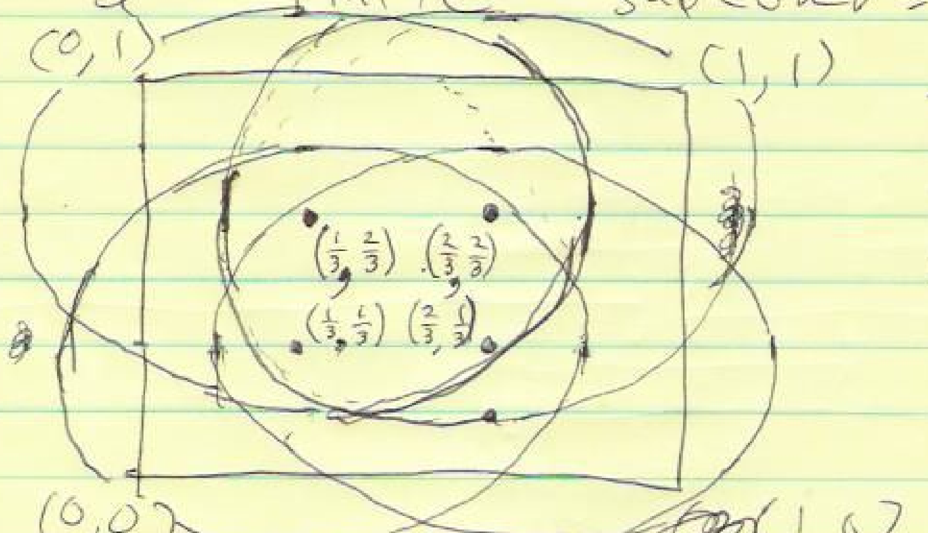
The sets $(\frac{1}{16}, \frac{1}{4}), (\frac{1}{32}, \frac{1}{8}), \ldots$ can be thrown out.

$X = \mathbb{R}^2 \quad E = [0,1]^2 \subset X$

E is compact. For example, the collection $\{B_{1/2}(p) : p \in \mathbb{R}^2\}$ of all open disks of radius $1/2$ covers E and

$$\left\{B_{1/2}\left(\tfrac{1}{3}, \tfrac{2}{3}\right), B_{1/2}\left(\tfrac{2}{3}, \tfrac{1}{3}\right), B_{1/2}\left(\tfrac{1}{3}, \tfrac{1}{3}\right), B_{1/2}\left(\tfrac{2}{3}, \tfrac{2}{3}\right)\right\}$$

is a finite subcover:



To prove this, you need $\frac{\sqrt{2}}{3} < \frac{1}{2}$ and some easy geometry.