Let X be a metric space and $E \subset X$. Let $a \in X$.

~~let Y also~~

- E is <u>open</u> iff $\forall p_1, p_2, p_3, \dots \in X$ $\left[\lim_{n\to\infty} p_n \in E \Rightarrow \underbrace{\text{eventually}}_{\exists N\ \forall n \geq N}\ p_n \in E\right]$
- E is <u>closed</u> iff $\forall p_1, p_2, p_3, \dots \in E$ $\left[\lim_{n\to\infty} p_n \in E \text{ or } \lim_{n\to\infty} p_n \text{ does not exist in } X.\right]$
- $a \in E'$ iff $\exists p_1, p_2, p_3, \dots \in E$ $\left[a = \lim_{n\to\infty} p_n \text{ and } \forall n \quad a \neq p_n\right]$.
- E is <u>perfect</u> iff $E = E'$.
- E is <u>dense</u> iff $\forall p \in X$ $\exists p_1, p_2, p_3, \dots \in E$ $\lim_{n\to\infty} p_n = p$.

- E is <u>compact</u> iff $\forall p_1, p_2, p_3, \ldots \in E$

  $\exists n_1 < n_2 < n_3 < \cdots \quad \lim_{k \to \infty} p_{n_k} \in E$

- E is <u>connected</u> iff

  $\forall f : E \to \{0,1\}$ [either f is constant

  or $\exists p_1, p_3, p_5, \ldots \in f^{-1}\{0\}$

  $\exists p_2, p_4, p_6, \ldots \in f^{-1}\{1\} \quad \lim_{n \to \infty} p_n \in E$]

Let Y also be a metric space

and $b \in Y$. Let $f : X \to Y$.

- $\lim_{x \to a} f(x) = b$ iff $\Big[\forall p_1, p_2, p_3, \ldots \in X$

  [if $a \neq p_1, p_2, p_3, \ldots$ and $\lim_{n \to \infty} p_n = a$,

  then $\lim_{n \to \infty} f(p_n) = b$] and $a \in X'\Big]$.

- f is <u>continuous</u> iff $\forall p_1, p_2, p_3, \ldots \in X$

  $\forall q \in X \left[\lim_{n \to \infty} p_n = q \Rightarrow \lim_{n \to \infty} f(p_n) = f(q) \right]$

- f is <u>continuous at a</u> iff

  $\forall p_1, p_2, p_3, \ldots \in X \left[\lim_{n \to \infty} p_n = a \Rightarrow \lim_{n \to \infty} f(p_n) = f(a) \right]$