Geometric series:

Assume $r \in \mathbb{R}$ and $n \in \mathbb{N}_0$.

Then $(1+r+r^2+\cdots+r^{n-2}+r^{n-1}+r^n)(1-r)$

$$=(1+r+r^2+\cdots\cdots\cdots+r^{n-2}+r^{n-1}+r^n)$$
$$-(r+r^2+r^3+\cdots\cdots\cdots+r^{n-1}+r^n+r^{n+1})$$

$$=1-r^{n+1}.$$

Assume also $r \neq 1$. Then $\sum_{k=0}^{n} r^k$

$$=1+r+r^2+\cdots+r^{n-2}+r^{n-1}+r^n=(1-r^{n+1})/(1-r)$$

If $|r|<1$, then $\lim_{n\to\infty} r^n=0$,

in which case $\lim_{n\to\infty}\sum_{k=0}^{n} r^k = \frac{1-0}{1-r}$

$$=(1-r)^{-1}.$$

If $a, b \in \mathbb{Z}$, $a < b$, and $1 \neq r \in \mathbb{R}$,

then $r^a + r^{a+1} + r^{a+2} + \cdots + r^{b-2} + r^{b-1} + r^b$

$$= r^a\left(1 + r + r^2 + \cdots + r^{b-a-2} + r^{b-a-1} + r^{b-a}\right)$$

$$= r^a\left(\frac{1 - r^{b-a+1}}{1-r}\right) = \frac{r^a - r^{b+1}}{1-r}.$$

If also $|r| < 1$, then $\sum_{k=a}^{\infty} r^k$

$$= \lim_{b\to\infty} \sum_{k=a}^{b} r^k = \lim_{b\to\infty} \frac{r^a - r^{b+1}}{1-r} = \frac{r^a}{1-r}$$