

Test II

MATH 4335

Instructor: David Milovich

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★ Name: _____

Hand-written notes are allowed;
attach to test when completed.

#	Pts possible	Pts achieved
1	25	
2	25	
3	25	
4	25	
Total	100	

#1 True or false:

Continuous images of compact sets are compact.

Continuous preimages of compact sets are bounded.

Continuous images of compact sets are bounded.

Continuous preimages of compact sets are closed.

At least one of the above is false.

Demonstrate this with a counterexample:

#2 Classify each series as divergent, absolutely convergent, or conditionally (i.e., non-absolutely) convergent.

$$\sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^n$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$$

$$\sum_{n=1}^{\infty} \frac{e^n}{n}$$

$$\sum_{n=1}^{\infty} \frac{n}{3^n}$$

$$\sum_{n=1}^{\infty} (-1)^n$$

#3 Circle one of these two statements and then prove it.

A) Every bounded sequence of reals has a convergent subsequence.

B) $3x + \sin(x)$ is a uniformly continuous function from \mathbb{R} to \mathbb{R} .

#4 Prove directly from the definition of limit that $\lim_{x \rightarrow 5} \frac{1}{x} = \frac{1}{5}$.

(Hint: $\frac{1}{x} - \frac{1}{5} = \frac{5-x}{5x}$.)

(By "definition of limit," I mean the ϵ - δ definition.)