MATH 4335 FINAL EXAM

Name: ______________________________

Instructions: Solve five of the seven problems. I will grade the first five problems for which I see a solution attempt that has not been crossed out.

Warning: For each problem, I want only your single best solution attempt, with any other proof attempts crossed out. If you do not heed my warning, I will grade based on the worst solution attempt.

Exercise	Grade
1	
2	
3	
4	
5	
6	
7	
Overall	

Date: Dec. 12, 2016.

1. Let $a_n = \left(1 + \frac{(-1)^n}{n}\right)^n$ for $n = 1, 2, 3, \ldots$

(1) Is the sequence $\{a_n\}$ monotone? Justify your answer.
(2) Is $\{a_n\}$ bounded? Justify your answer.

2. Let $f(x) = 1/(1+x^2)$ for all $x \in \mathbb{R}$.

(1) Is f bounded on $\mathbb{R}$? Justify your answer.

(2) Is f *locally* monotone on $\mathbb{R}$? Justify your answer.

3. Use the inverse function rule for derivatives to prove that $(x^{1/n})' = \frac{1}{n}x^{(1-n)/n}$ for all $n \in \{1, 2, 3, \ldots\}$ and $x \in (0, \infty)$. (You may assume the power rule $(x^m)' = mx^{m-1}$, but only for $m \in \{1, 2, 3, \ldots\}$. The purpose of the exercise is to prove the power rule for a class of fractional exponents.)

4. Given $I_0 = [2, 3]$ and $f(x) = x^3 + x - 18$, use bisection to find a subinterval I_3 of I_0 such that I_3 has width $1/8$ and $f(c) = 0$ for some $c \in I_3$.

5.

(1) Is there a sequence $\{a_n\}$ such that $a_n \to 0$ and $\sum_{n=1}^{\infty} a_n$ diverges? Give an example or prove that there is no example.

(2) Is there a sequence $\{a_n\}$ such that $a_n \to \frac{1}{2}$ and $\sum_{n=1}^{\infty} a_n$ converges? Give an example or prove that there is no example.

6. Let $f(x) = \sum_{n=0}^{\infty}(x/3)^n$. Without using logarithms, prove that f is well-defined and integrable on $[0, 1]$, and that $\int_0^1 f(x)\,dx \geq \frac{7}{6}$.

7. Let B be the Bolzano-Weierstrass Theorem, F be the two Fundamental Theorems of Calculus, I be the integrability of continuous functions, and U be the Uniform Continuity Theorem. Which statement below is most accurate? (No proof required.)

(1) B helps prove F; F helps prove I; I helps prove U.
(2) B helps prove I; I helps prove U; U helps prove F.
(3) B helps prove I; I helps prove F; F helps prove U.
(4) B helps prove U; U helps prove I; I helps prove F.
(5) U helps prove F; F helps prove I; I helps prove B.
(6) U helps prove B; B helps prove F; F helps prove I.
(7) U helps prove I; I helps prove F; F helps prove B.
(8) U helps prove F; F helps prove B; B helps prove I.