3.2 #6 a) False: $(-\infty, \sqrt{2}) \cup (\sqrt{2}, \infty)$.

b) False: $\bigcap_{n=1}^{\infty} [n, \infty) = \phi$

c) True; given $a \in O$ open, we have some $\varepsilon > 0$ with $(a-\varepsilon, a+\varepsilon) \subset O$; there are rationals strictly between $a-\varepsilon$ & $a+\varepsilon$.

d) False: $\{\sqrt{2}/n : n \in \mathbb{N}\}$

e) True: The Cantor set is $\bigcap_{n=1}^{\infty} C_n$ where each C_n is a union of 2^n-many closed intervals (page 85).

By Theorem 3.2.14(i), each C_n, being a finite union of closed sets, is closed.

By 3.2.14(ii), $\bigcap_{n=1}^{\infty} C_n$, being an intersection of closed sets, is closed.

(Given: A open & B closed.)

3.2#8 a) Definitely closed. (See Theorem 3.2.12.)

b) Definitely open. (See 3.2.13, 3.2.3.)

$$\Big(\underbrace{A}_{\text{open}} \setminus \underbrace{B}_{\text{closed}} = \underbrace{A}_{\text{open}} \cap \underbrace{B^c}_{\text{open}}\Big)$$

c) Definitely open. (See 3.2.13, 14)

$$\underbrace{\Big(\underbrace{\underbrace{\underbrace{A}_{\text{open}}{}^{c}}_{\text{closed}} \cup \underbrace{B}_{\text{closed}}}_{\text{closed}}\Big)^c}_{\text{open}}$$

d) $(A \cap B) \cup (A^c \cap B) = B$ is definitely closed.

(Why? $(A \cap B) \cup (A^c \cap B)$
$= \underbrace{\{x \in B : x \in A\}} \cup \{x \in B : x \notin A\}$.)

Reads "the set of all x in B such that x is in A."

e) $\overline{A}^c \cap \overline{A^c} = \overline{A}^c$ is definitely open.
(Why? $\overline{A}$ is closed, so $\overline{A}^c$ is open.
Moreover, $\overline{A}^c \subseteq \overline{A^c}$ because $A^c \subseteq \overline{A^c}$ and
$\overline{A}^c \subseteq A^c$ (because $\overline{A} \supseteq A$).)