8-31-10 Topology

Recall $f: A \to B$ means $f \subseteq A \times B$, f is a rule of assignment, (domain F) $= A$, and (domain f) $\subseteq B$.

$(a,b) \neq (b,a)$ [ordered pairs]
unless $a=b$

$\{a,b\} = \{b,a\}$ [unordered pairs]

Relations (section 3) are more general than functions.

"R is a relation on A" means $R \subseteq A \times A$.

$$\begin{cases} f(a)=b \Leftrightarrow (a,b) \in f \\ aRb \Leftrightarrow (a,b) \in R \end{cases}$$

In an ordered pair, order matters.

$A \times B = \{(a,b) \mid a \in A \ b \in B\}$

Is $A \times B$ a relation a $A \cup B$?

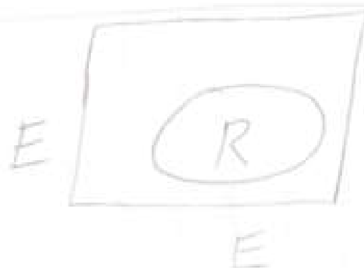
asking

Is $A \times B \subseteq (A \cup B) \times (A \cup B)$?

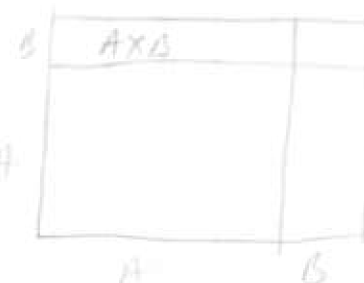
yes.

If $(a,b) \in A \times B$, then $a \in A \subseteq A \cup B$, so $a \subseteq A \cup B$. Also $b \in B \subseteq A \cup B$, so $b \in A \cup B$. So, $(a,b) \in (A \cup B) \times (A \cup B)$

$R \subseteq E \times E$



$A \times B \subseteq (A \cup B) \times (A \cup B)$



Proof by picture

A relation R on A is an equivalence relation if:

1) (reflexivity) - everything related to itself $a \in A \Rightarrow aRa$

2) (Transitivity) - $(aRb \text{ and } bRc) \overset{\Rightarrow}{\text{then}} aRc$ $x=y$ $y=z$ $x=z$

3) (Symmetry) - is a sibling of $aRb \Rightarrow bRa$

Example

$(A \times A) \cup (B \times B)$ is an equivalence relation on $A \cup B$ if $A \cap B = \emptyset$.

either both in A or both in B

$A \times B$ is empty = no alike terms in both A and B.

Particians

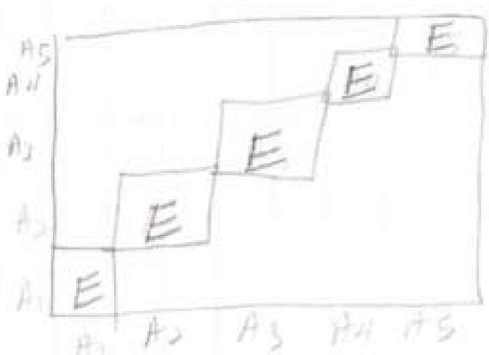
D is a Partician of A if

$D \subseteq P(A) - \{\emptyset\}$, - everything in D is subset of A

$X, Y \in D$ and $X \neq Y \Rightarrow X \cap Y = \emptyset$,

and $\underbrace{\bigcup D}_{\text{union of } D} = A$

$\{A_1, A_2, A_3, A_4, A_5\}$ is a Partician of A.



If E is equivalence relation and

$x \in a/E \cap b/E$, then

$a E x E b$, so $a E b$, so

$a/E = b/E$.

NOT SURE
Need to check last part.

contrapositive - if they are not equal then they are disjoint.

Equivalence Relation

Say $A=[0,2]$ and $B=[1,3]$.

$R=(A\times A)\cup(B\times B)$.

$(\frac{1}{2},\frac{3}{2})\in R\ [\Leftrightarrow \frac{1}{2} R \frac{3}{2}]$

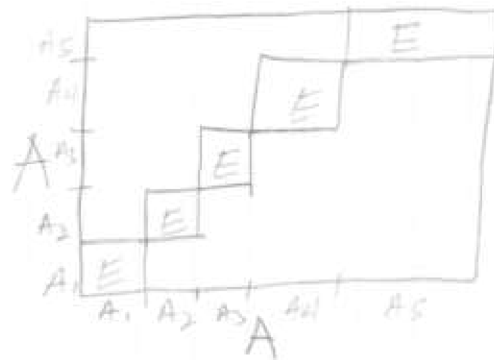
ordered pair

$(\frac{3}{2},3)\in R\ [\Leftrightarrow \frac{3}{2} R\, 3]$

But $(\frac{1}{2},3)\notin R$

not in B not in A

E is an equivalence relation on A.



If E is an equivalence relat
on A and $a\in A$,
then $\{x \mid xEa\}$
is called the
Equivalence class of
E, written

a/E.

Order

$R \subseteq A \times A$ is an order Relation if

- (Nonreflexivity) $a \in A \Rightarrow (a,a) \notin R$
  opposite of reflexivity
  not just the absence
- (Transitivity) $aRbRc \Rightarrow aRc$
- (Comparability) ($a, b \in A$ and $a \neq b$) $\Rightarrow$ (aRb or bRa)
  ($x<y$ or $y<x$)

Picture:

generic symbols:

$\sim$ for equivalence relation

$<$ for order relation.

Open Interval $(a,b) = \{x \mid a<x<b\}$

makes sense for any order relation $<$.

If $(a,b) \neq \emptyset$, then (picture 

a is the immediate predicessor of b, or

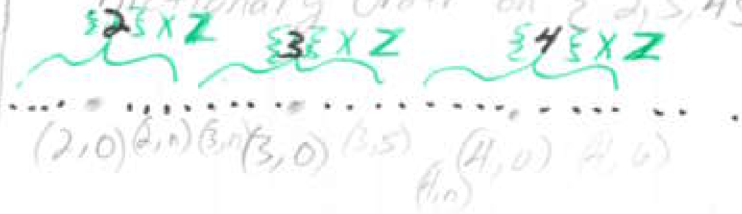
b is the immediate successor of a.

Dictionary Order:

If $<_A$, $<_B$ under A, B (respectively) then the dictionary order $<$ on $A \times B$ is defined by:

$$(a_1, b_1) < (a_2, b_2) \Leftrightarrow \left(a_1 <_A a_2 \text{ or } (a_1 = a_2 \text{ and } b_1 <_B b_2)\right).$$

Dictionary Order on $\{2,3,4\} \times \mathbb{Z}$



As a suborder of $\mathbb{R}$:

$$\left\{ x + \frac{2}{\pi} \arctan(y) : x \in \{0,2,4\}, y \in \mathbb{Z} \right\}$$

Everything has an immediate predecessor & successor.

If $<$ is an order relation on A, and $B \subseteq A$, then $\max(B)$ is the greatest element of B (if it exists).

(Likewise define $\min(B)$)

$$\mathrm{Sup}(B) = \min(\{x \mid \underbrace{b \overset{< \text{ or } =}{\leq} x \text{ for all } b \in B}_{} \})$$

Supremum of B is least upper bound.

least upper bound of B

An order relation $<$ has the least upper bound property if every nonempty set that has an upper bound has a least upper bound