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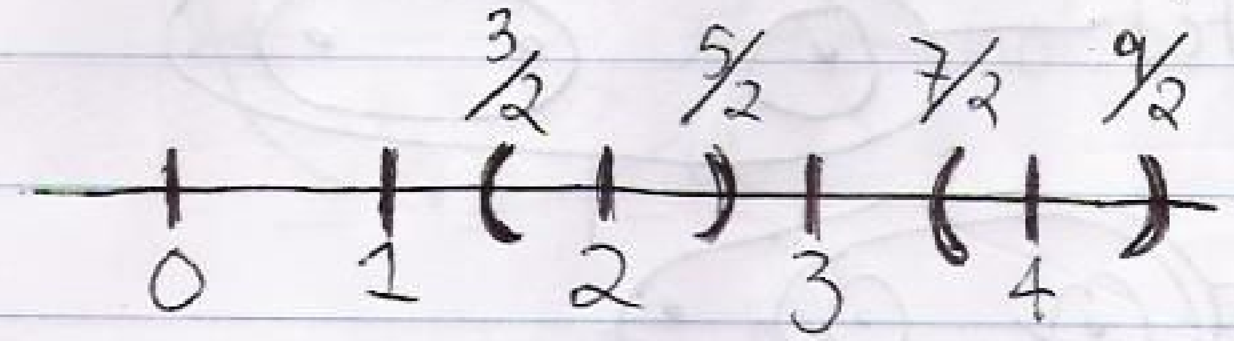
$\mathbb{Q} = (\mathbb{Q} \cap \underbrace{(-\infty, \sqrt{2})}_{\text{open in } \mathbb{R}}) \cup (\mathbb{Q} \cap \underbrace{(\sqrt{2}, \infty)}_{\text{open in } \mathbb{R}})$

$\mathbb{Z} =$

Prove that $A = \{2n / n \in \mathbb{Z}\}$ is open in $\mathbb{Z}$.

subspace of $\mathbb{R}$



$A = \mathbb{Z} \cap \bigcup_{n \in A} (n - 1/2, n + 1/2)$

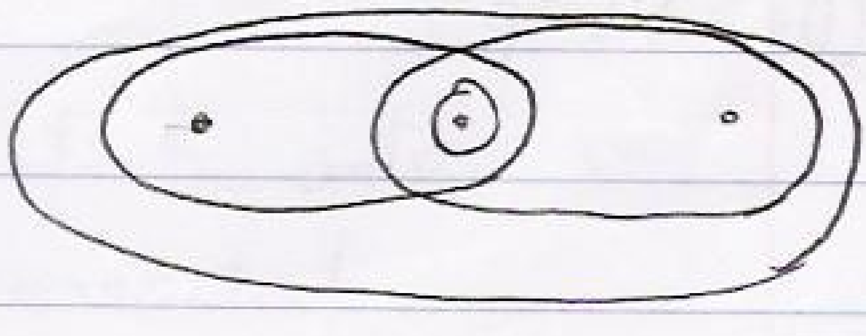
$A = \mathbb{Z} \cap \bigcup_{n \in A} (n - 1, n + 1)$

$A = \mathbb{Z} \cap \bigcup_{n \in A} (n - 1/10, n + 1/10)$

(It works for any $A \subset \mathbb{Z}$! :))

$(X, \mathcal{T}_X) = (\{1,2,3\}, \{\emptyset, \{1,2,3\}, \{1,2\}, \{2\}, \{2,3\}\})$

is connected



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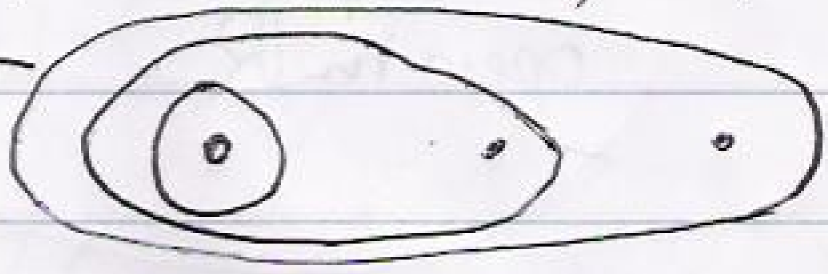
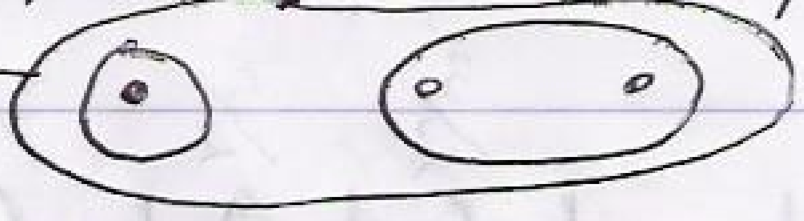
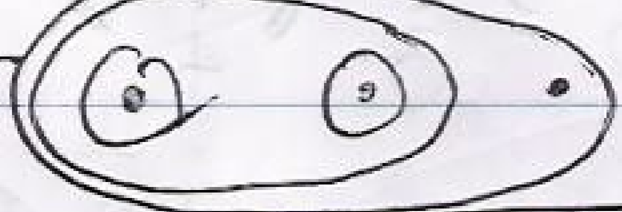
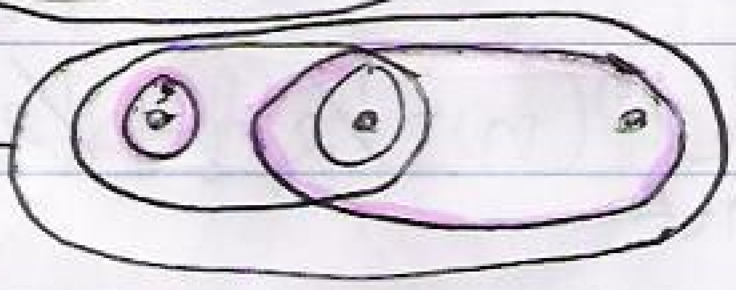
$(Y, \mathcal{T}_Y) = (\{1,2,3\}, \{\emptyset, \{1,2,3\}, \{1\}, \{1,2\}\})$

is connected – 

$(Z, \mathcal{T}_Z) = (\{1,2,3\}, \{\emptyset, \{1,2,3\}, \{1\}, \{2,3\}\})$

is NOT connected – 

connected – 

not connected – 

To be proved in §24:

Every real interval is connected.

$f: \mathbb{R} \to S^1 = \{(x,y) \in \mathbb{R}^2 \mid x^2+y^2=1\}$

$f(t) = (\cos(t), \sin(t))$

is continuous and is onto.

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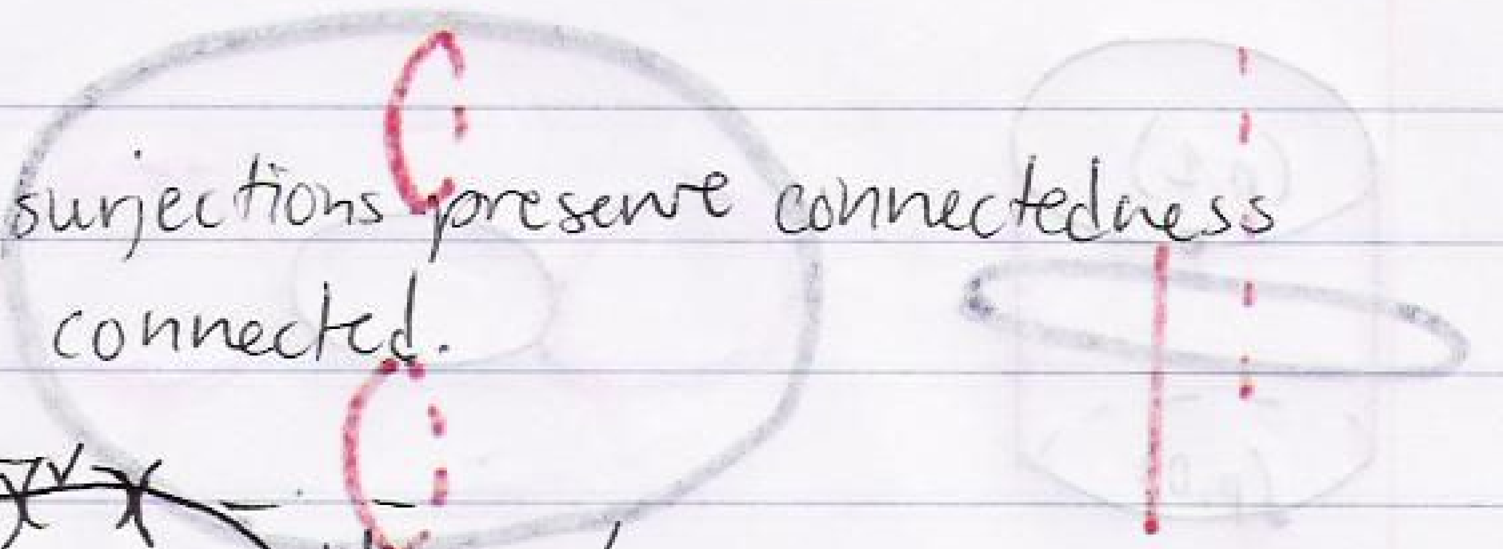
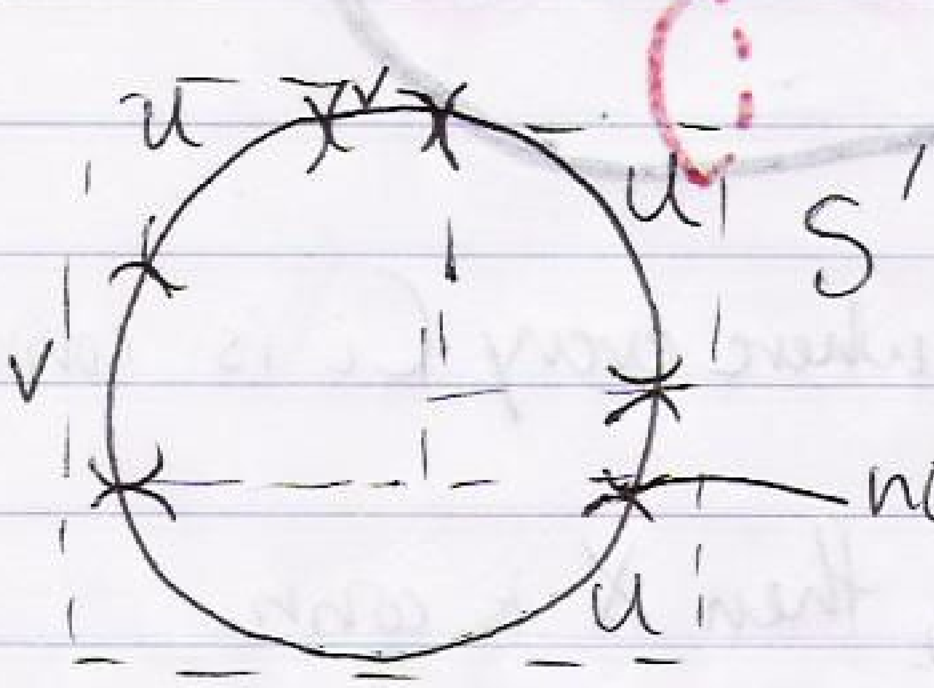
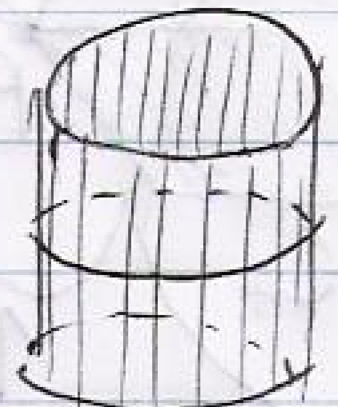


Continuous surjections preserve connectedness
So, S^1 is connected.



Products preserve connectedness

$$\overbrace{S^1}^{\text{conn.}} \times \overbrace{[0,1]}^{\text{conn.}}$$



$\Rightarrow S^1 \times [0,1]$ is conn.

the torus/donut

$p: S^1 \times [0,1] \to \mathbb{T}^2$ is the quotient map induced by equivalence relation

$(p,q) \sim (r,s) \Longleftrightarrow (p = r \,\&\, (q = s \text{ or } (q = 0 \,\&\, s = 1) \text{ or } (q = 1 \,\&\, s = 0)))$

quotient maps are cts & onto, so $\mathbb{T}^2$ is connected

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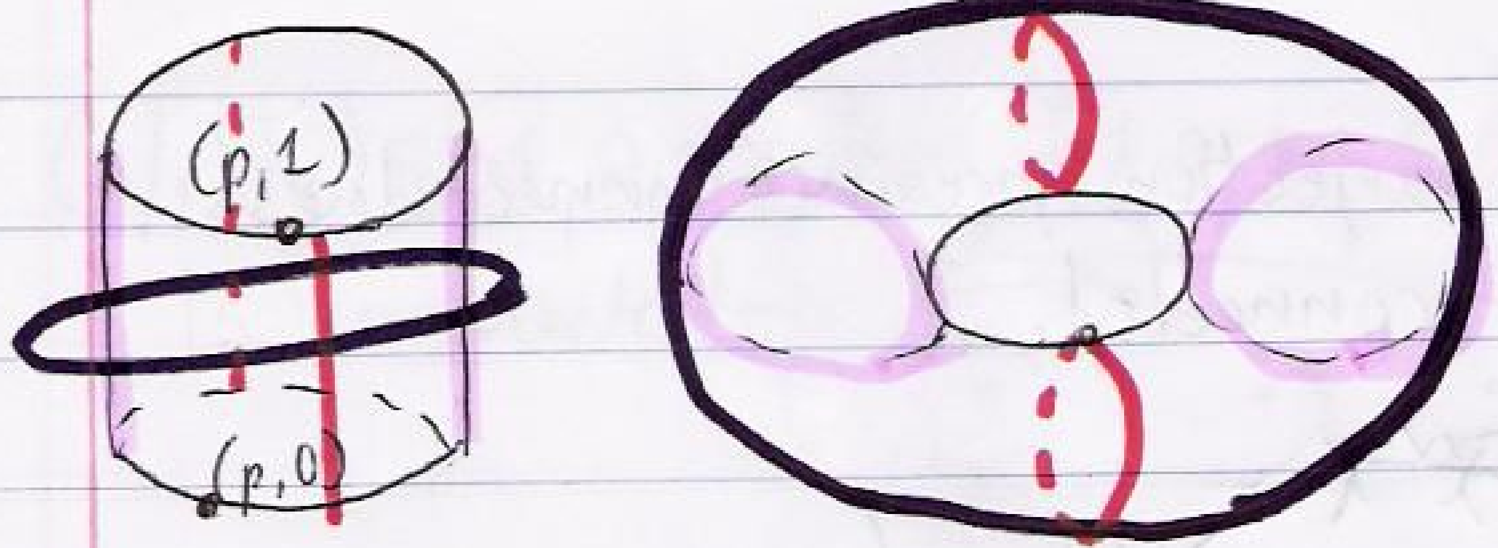
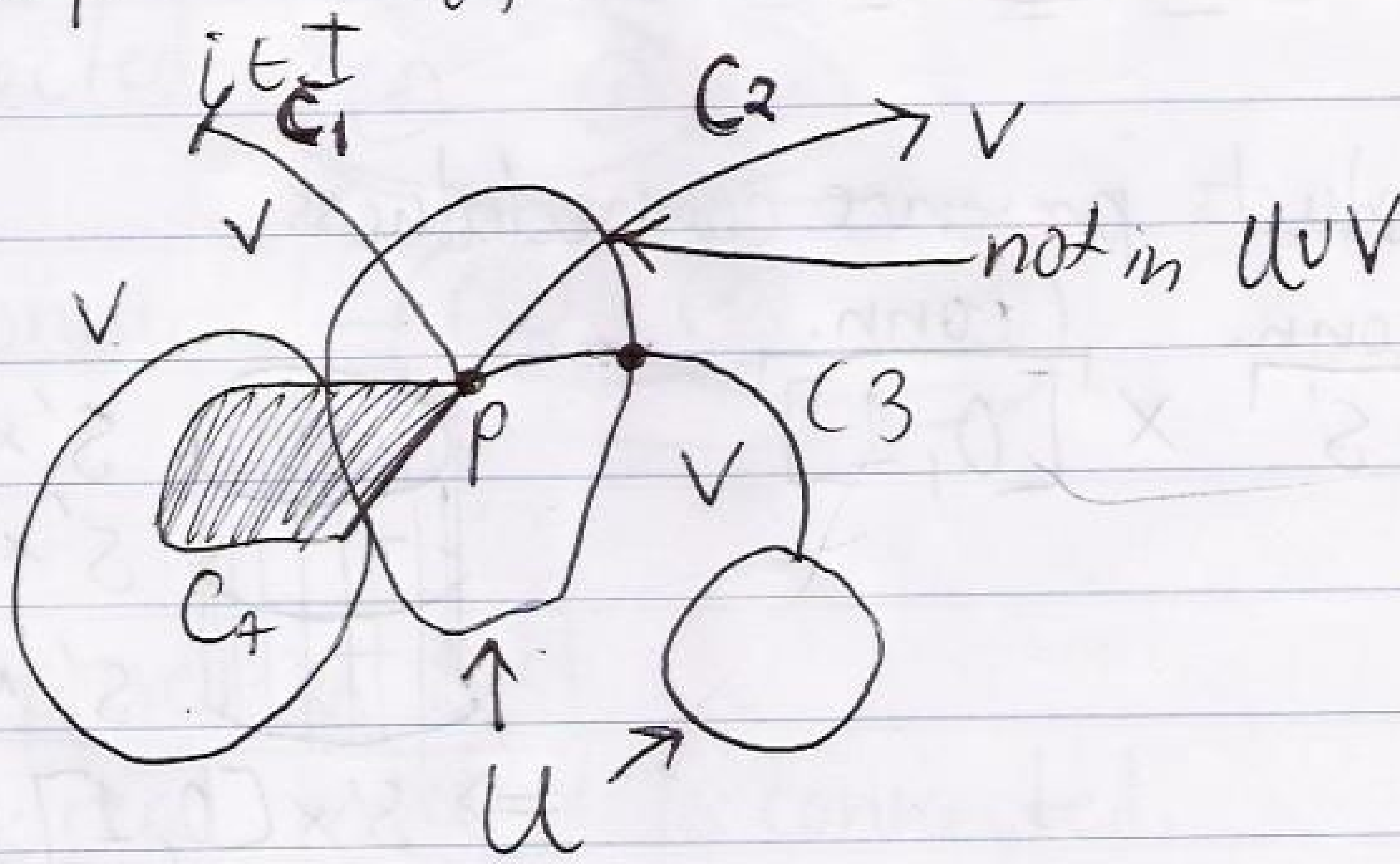


If $X = \bigcup_{i \in I} C_i$ where every C_i is conn.

& $\exists\, p \in \bigcap_{i \in I} C_i$, then X is conn.



If $A \subset X$ & A is conn. (in the subspace topology), then, if $A \subset B \subset \overline{A}$, B is also be connecte