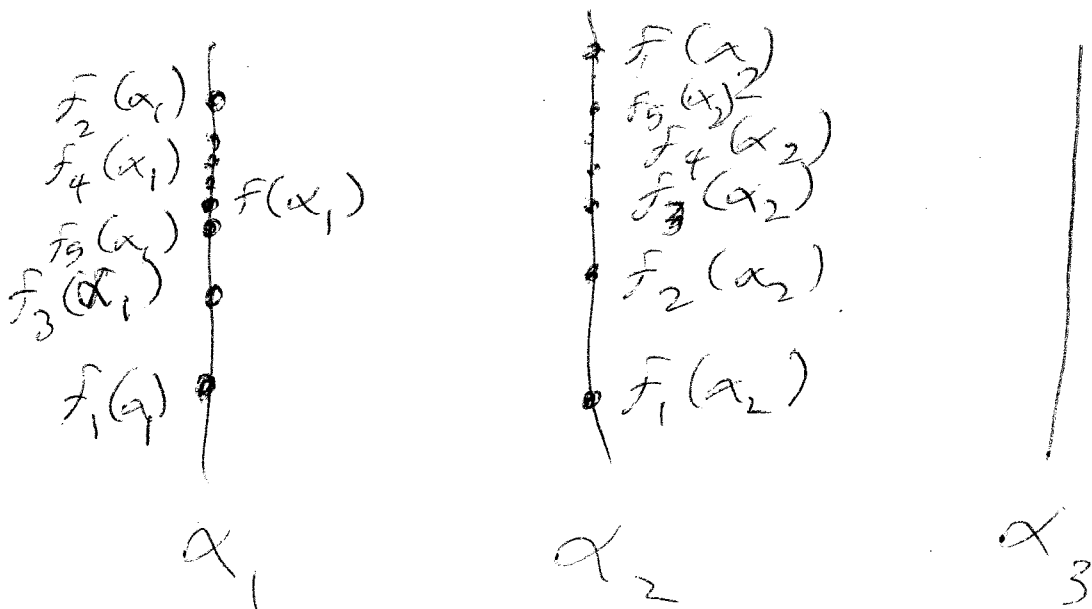


$f_1, f_2, f_3, \dots \rightarrow f$  in  $\alpha$

product topology.  $X = \prod_{\alpha \in J} X_\alpha$

iff  $\forall \alpha \in J \quad f_1(\alpha), f_2(\alpha), f_3(\alpha), \dots$

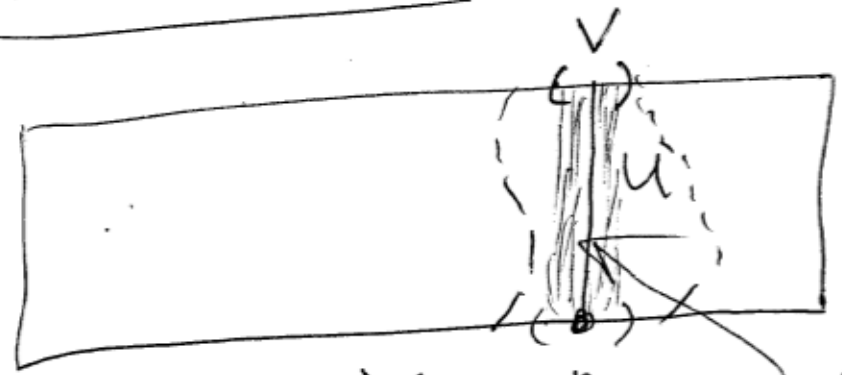
$\rightarrow f(\alpha)$



Convergence of sequences  
in a product top. is  
pointwise. (43.3)

Tube Lemma (§ 26.8)

Used to prove preservation of compactness by finite products without the Axiom of Choice. (§ 25)



$X \quad p \quad \{p\} \times Y \subset U \text{ open} \subset X \times Y$

Given  $p, U, X, Y$  as above, there is an open  $V \subset X$  with  $p \in V$  &  $V \times Y \subset U$ .



$Y = [0, 1) \quad \{0\} \times [0, 1) \subset U$   
 $U = \{(x, y) \in X \times Y \mid y < 1 - |x|\}$   
 If  $0 \in V \text{ open} \subset [-1, 1]$ ,  
 then  $V \times [0, 1) \not\subset U$ .

So, compactness must be used in the proof of the Tube Lemma.

Property	cts image	closed subspace	products
compact	yes	yes	yes
conn.	yes	no $\{0,1\} \subset \mathbb{R}$	yes
path conn.	yes	no	yes
Seq cpet.	yes	yes	yes for <del>compact</del>

yes for  
~~compact~~  
ctbl prod's

↳ Preservation?

INDISCRETE  
TOPOLOGY

YES

- T<sub>1</sub>
- T<sub>2</sub>
- T<sub>3</sub>
- T<sub>3,5</sub>
- T<sub>4</sub>
- T<sub>5</sub>
- T<sub>6</sub>

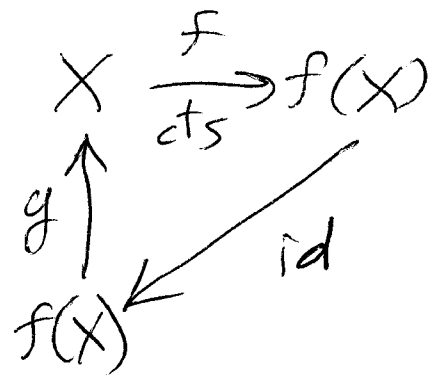
- yes
- yes
- yes
- yes
- no
- no

yes for  
ctbl prod's

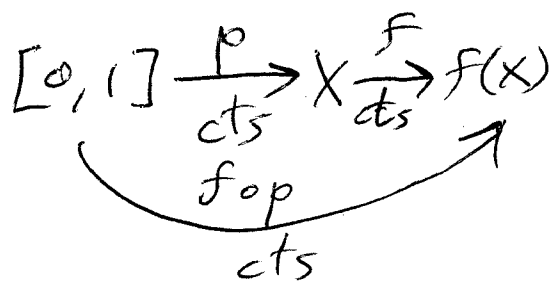
Path. conn.  
preservation  
by products:  
use diagonal  
product of paths.

Let  $x_n \in f(X)$   
&  $x_n = f(g(x_n))$   
&  $g(x_{k_n}) \rightarrow L$   
Then  $x_{k_n} \rightarrow f(L)$ .

Preservation  
of seq. cpts.  
by its images:  
proof ideas.



Path. conn.  
preservation  
by cts. images:



Compose with paths.

Preservation of  
seq. cpts. by  
closed subspaces:

~~$x_n \in C$~~   $x_n \in C = \bar{C} \subset X$   
 $\Rightarrow x_{n_k} \rightarrow L \in \bar{C} = C$