Day 5 HW due day 10.

① [Grad. only] Prove that if $\mathcal{S}_X$ and $\mathcal{S}_Y$ are subases for topologies $\mathcal{T}_X$ and $\mathcal{T}_Y$ on X and Y, and $A \subset X \times Y$, then $\{A \cap \pi_1^{-1}(U) \mid U \in \mathcal{S}_X\} \cup \{A \cap \pi_2^{-1}(V) \mid V \in \mathcal{S}_Y\}$ is a subbase for the subspace topology A inherits from the product space.

② Let $X = \{1, 2, 3, 4\}$ and let $\mathcal{B}_X = \{\{1\}, \{2\}, \{2,3,4\}\}$ be a base for topology $\mathcal{T}_X$ on X. Find a base for the subspace $A = \{(x,y) \in X^2 \mid x \leq y\}$ of the product space X^2.

③ Let $\mathbb{R}_\ell$ be $\mathbb{R}$ with the <u>lower limit topology</u>, which has base $\{[a,b) \mid a<b \text{ in } \mathbb{R}\}$. Prove that the subspace $D = \{(x,y) \mid x+y=0\}$ of $\mathbb{R}_\ell^2$ is discrete. Hint: prove that $\forall d \in D$ $\{d\}$ is open in D.

[Grad. only]

(4) Prove that if X has an order topology and A is a subset of X, then the order topology on A is coarser than the subspace topology on A. Hint: Prove that the order top. on A has a subbase that is a subset of a subbase of the subspace top. (Do <u>not</u> assume A is convex.)