HW6

① Find intervals A, B, C, D of reals such that $\overline{A} \cap \overline{B} \not\subset \overline{A \cap B}$ and $\overline{C - D} \not\subset \overline{C} - \overline{D}$. Then find a sequence of intervals $E_1, E_2, E_3, \cdots$ of reals such that $\bigcap_{n \in \mathbb{N}} \overline{E_n} \not\subset \overline{\bigcap_{n \in \mathbb{N}} E_n}$. (All closures are meant to be computed in $\mathbb{R}$ for this problem.)

② Let $Z = \{(x,y) \mid 0 < y < x \leq 1\} \subset Y = (0,1]^2 \subset X = \mathbb{R}^2$. Sketch Z & Y. Then compute $Cl_X Z$, $Int_X Z$, $Cl_Y Z$, and $Int_Y Z$. (These 4 sets are all different.)

③ The set A' of limit points of a subset A of a space X is $A' = \{p \in X \mid p \in Cl_X(A - \{p\})\}$. Let X be $\mathbb{N}^2$ with the dictionary order topology and $A = \{(m,n) \in X \mid m \geq 2\}$. Compute A' and $A'' = (A')'$.

(4) [Grad only] Prove $\overline{A} - \overline{B} \subset \overline{A - B}$.
(Assume only that X is a space and $A, B \subset X$.)

(5) Find a base for a topology on $\{1, 2, 3, 4\}$
such that the sequence $1, 2, 1, 2, 1, 2, 1, 2, 1, 2, \ldots$
converges to 2 and to 3 but not to 4
and not to 1.

Note: When I don't use words like
prove/explain/justify/why/show, then
a proof is not required. For this homework,
only (4) needs a proof.