MATH 4360 FINAL

INSTRUCTOR: DAVID MILOVICH

Name: _____

Date: Thursday, December 9, 2010.

Exercise	Point Possible	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

1. [20 points]

(a) Give an example of a bounded, connected subspace of $\mathbb{R}$ that is not compact.

(b) Give an example of a compact space that is countably infinite.

2. [20 points] Consider $A = [0, 1) \cup (1, 2] \cup \{3\}$ as a subset of $\mathbb{R}$.

(a) What is the closure of A?

(b) What is the set of limit points of A?

3. **[20 points]** Argue informally that a circle ($\circ$) is not homeomorphic to a figure-eight (∞).

4. **[20 points]** Let X be $\mathbb{Z}$ with the finite complement topology. Prove that X is not Hausdorff.

5. [20 points] Let $X = \prod_{n \in \mathbb{Z}_+} \mathbb{R}$. Define a sequence $(x_n)_{n \in \mathbb{Z}_+}$ in X by $x_n(m) = m/n$.

(a) Does the sequence converge in the product topology? If it does converge, then what does it converge to?

(b) Does the sequence converge in the uniform topology? If it does converge, then what does it converge to?

(c) Does the sequence converge in the box product topology? If it does converge, then what does it converge to?