MATH 4360 MIDTERM I

INSTRUCTOR: DAVID MILOVICH

Name: ______________________________

Date: Tuesday, September 21, 2010.

Exercise	Point Possible	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

1. [20 points] For each of the following four statements S, is S true or false?

(a) $A \cap (C \cup (A \cap B)) = A \cap (B \cup C)$

(b) $A - C \subset A - B \Rightarrow B \subset C$

(c) $B \subset C \Rightarrow A - C \subset A - B$

(d) $(A \times C) \cap (B \times A) = (A \cap B) \times (A \cap C)$

2. [20 points]

$$E = \{(1,1),(2,2)\} \cup \{(x,y) \in \mathbb{Z}_+ | x, y \geq 3\}$$

Give a precise description of each E-equivalence class.

3. [20 points] Give an example of a pair of nonempty subsets A, B of $\mathbb{Q}$ (where $\mathbb{Q}$ is the set of all rational numbers) for which the following three statements are all simultaneously true of A and B.

- A and B both have an upper bound in $\mathbb{Q}$.
- A has a least upper bound in $\mathbb{Q}$.
- B does not have a least upper bound in $\mathbb{Q}$.

4. [20 points] Give an example of nonempty sets A, B_1, B_2 and a function $f\colon A \to B_1 \times B_2$ such that f is not surjective, but both $\pi_1 \circ f$ and $\pi_2 \circ f$ are surjective.

Here, π_1 and π_2 are functions $\pi_1\colon B_1 \times B_2 \to B_1$ and $\pi_2\colon B_1 \times B_2 \to B_2$ defined by $\pi_1(b_1, b_2) = b_1$ and $\pi_2(b_1, b_2) = b_2$.

5. [20 points] Prove the following theorem by induction.

Theorem. If $n \in \mathbb{Z}_+$, the sets $A_1, A_2, \ldots, A_{n+1}$ are nonempty, and, for all $i \in \{1, \ldots, n\}$, there is an injection from A_i to A_{i+1}, then there is a surjection from A_{n+1} to A_1.