## MATH 4360 MIDTERM II

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Name:

Exercise	Point Possible	Score
1	42	
2	15	
3	28	
4	15	
Total	100	
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1. [42 points] Fill in each blank with "A," "B," "C," or "D" as appropriate.
Let $X_1 = \mathbb{R}$ with the standard topology. [2,3) is in $X_1$ .  A) open, but not closed B) closed, but not open C) open and closed D) neither open nor closed
Let $X_2 = \mathbb{R}$ with the discrete topology. [2,3) is in $X_2$ .  A) open, but not closed B) closed, but not open C) open and closed D) neither open nor closed
Let $X_3 = [2, 4]$ with the subspace topology from $X_1$ . $[2, 3)$ is in $X_3$ . A) open, but not closed B) closed, but not open C) open and closed D) neither open nor closed
Let $X_4 = [1, 3)$ with the subspace topology from $X_1$ . $[2, 3)$ is in $X_4$ . A) open, but not closed B) closed, but not open C) open and closed D) neither open nor closed
Let $X_5 = [1,3) \cup [4,5]$ with the subspace topology from $X_1$ . $[2,3)$ is in $X_5$ . A) open, but not closed B) closed, but not open C) open and closed D) neither open nor closed
Let $X_6 = [1,3) \cup [4,5]$ with the order topology. $[2,3)$ is in $X_6$ .  A) open, but not closed B) closed, but not open C) open and closed D) neither open nor closed

**2.** [15 points] Let  $X = \{1, 2\}$  with the topology  $\mathcal{T}_X = \{\emptyset, \{1\}, \{1, 2\}\}$ . Let  $Y = \{3, 4\}$  with the topology  $\mathcal{T}_Y = \{\emptyset, \{3\}, \{4\}, \{3, 4\}\}$ . Give an example of a basis  $\mathcal{B}$  for the product topology on  $X \times Y$ . Explicitly list the elements of your example  $\mathcal{B}$ .

3. [28 points] Let  $X = \{1, 2, 3\} \times (\{0\} \cup \mathbb{Z}_+)$ .

- (a) Prove that X is countable.
- (b) Prove that X with the dictionary order is well-ordered.

(You may assume that  $\mathbb{Z}_+$  is countable and well-ordered.)

**4.** [15 points] Give an example of a 6-element topology  $\mathcal{T} = \{U_1, U_2, U_3, U_4, U_5, U_6\}$  on the set  $X = \{1, 2, 3\}$ . Explicitly list the elements of your example  $\mathcal{T}$ .