MATH 5365 FINAL

INSTRUCTOR: DAVID MILOVICH

Name: ______________________________

Please sign to certify that:

- My answers to this exam are my own work.
- I may have consulted pre-existing textual and/or electronic resources, but,
- between the time I received this exam and the time I submitted my answers to my instructor, I have not directly nor indirectly asked other humans questions relevant to this exam.

Signature:__________________________________

Date: May 9, 2012.

Exercise	Point Possible	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

1. [20 points] Let $A = [1, 2)$ and $B = \{3 - 10^{-n} : n \in \mathbb{Z}_+\}$. (You don't need to prove anything for this exercise; just answer correctly.)

(a) What is the closure of $A \cup B$ in $\mathbb{R}$?
(b) What is the closure of $A \times B$ in $\mathbb{R} \times [2, 3)$?

2. [20 points] Define a sequence $(x_n)_{n\in\mathbb{Z}_+}$ in $\mathbb{R}^\omega$ by $x_n(m) = \frac{1}{n} - \frac{1}{m}$, that is,

$$x_n = (\frac{1}{n} - \frac{1}{1}, \frac{1}{n} - \frac{1}{2}, \frac{1}{n} - \frac{1}{3}, \ldots).$$

(You don't need to prove anything for this exercise; just answer correctly.)

(a) Does $(x_n)_{n\in\mathbb{Z}_+}$ converge in $\mathbb{R}^\omega$ with the product topology? If it does converge, then what does it converge to?

(b) Does $(x_n)_{n\in\mathbb{Z}_+}$ converge in $\mathbb{R}^\omega$ with the uniform topology? If it does converge, then what does it converge to?

(c) Does $(x_n)_{n\in\mathbb{Z}_+}$ converge in $\mathbb{R}^\omega$ with the box product topology? If it does converge, then what does it converge to?

3. [20 points] Let X be a Hausdorff space and let $C_1 \supset C_2 \supset C_3 \supset C_4 \supset \cdots$ be a descending chain of compact connected subspaces of X. Prove that the subspace $C = \bigcap_{n \in \mathbb{Z}_+} C_n$ is connected.

4. [20 points] Let $(X, \mathcal{T}_X)$ and $(Y, \mathcal{T}_Y)$ be T_3 spaces and assume X and Y are disoint sets. Give $X \cup Y$ the topology $\{U \cup V : U \in \mathcal{T}_X \text{ and } V \in \mathcal{T}_Y\}$. Prove that $X \cup Y$ is T_3.

5. [20 points] Let $X = [0,1]^\omega$, the set of all functions from $\mathbb{Z}_+$ to $[0,1]$. Prove that X with the uniform metric is complete but not compact.