MATH 4360 MIDTERM II

INSTRUCTOR: DAVID MILOVICH

Name: ________________________________

Date: Tuesday, October 19, 2010.

Exercise	Point Possible	Score
1	25	
2	25	
3	25	
4	25	
Total	100	

1. [25 points] Let $X = \mathbb{R}$ and let $Y = [0, 2) \cup (3, 5]$ with the subspace topology. Let $A = [1, 2) \cup (4, 5]$.

(a) What is the closure of A in X?
(b) What is the interior of A in X?
(c) What is the closure of A in Y?
(d) What is the interior of A in Y?

2. [25 points]

(a) Giving the set $\mathbb{Q}$ of all rational numbers the subspace topology inherited from $\mathbb{R}$, is $\mathbb{Q}$ a Hausdorff (*i.e.*, T_2) space?

(b) Give an example of a different topology on $\mathbb{Q}$ that is finer than the subspace topology inherited from $\mathbb{R}$.

3. [25 points] Let $X = \{1, 2, 3\}$ with the topology generated by the base $\mathcal{B}_X = \{\{1, 2\}, \{3\}\}$. Let $Y = \{4, 5, 6, 7\}$ with the topology generated by the base $\mathcal{B}_Y = \{\{4\}, \{5\}, \{5, 6, 7\}\}$. Give an example of a continuous injection (*i.e.*, one-to-one) from X to Y.

4. [25 points] Consider $X = \prod_{n \in \mathbb{Z}_+} [0,1]$ (with the product topology). Also consider the sequence $(x_n)_{n \in \mathbb{Z}_+}$ in X where $x_n(m) = 1$ if $m = n$ and $x_n(m) = 1/m$ if $m \neq n$. E.g., $x_1 = (1, 1/2, 1/3, 1/4, 1/5, \ldots)$, $x_2 = (1, 1, 1/3, 1/4, 1/5, \ldots)$, $x_3 = (1, 1/2, 1, 1/4, 1/5, \ldots)$, $x_4 = (1, 1/2, 1/3, 1, 1/5, \ldots), \ldots$.

(a) What is the unique element of X that $(x_n)_{n \in \mathbb{Z}_+}$ converges to?

(b) Prove that $(x_n)_{n \in \mathbb{Z}_+}$ does not converge to any point in X with the uniform topology induced by the standard metric $d(x, y) = |x - y|$ on $[0, 1]$.

5. [25 Extra Credit points] Prove that $X = \prod_{n \in \mathbb{Z}_+} [0, 1]$ with the uniform topology is not first countable.