MATH 5365 FINAL EXAM

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Name: ______________________________

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1. [25 points] Use a connectedness argument to prove that $\mathbb{R}$ is not homeomorphic to $\mathbb{R}^2$.

2. [30 points] Let $X = \prod_{n=1}^{\infty} \mathbb{R}$ with the product topology. Let $A = \bigcup_{n=1}^{\infty} \bigcap_{m=n}^{\infty} \pi_m^{-1}(\{0\})$, the set of all $p \in X$ such that $p(n)$ is eventually 0. Prove or disprove: A is dense in X.

3. **[45 points]** Suppose that (X, d) is a compact metric space and $f \colon X \to X$ is an isometric function (*i.e.*, $d(f(x), f(y)) = d(x, y)$ for all points $x, y \in X$).

(a) Prove that f is surjective. (Hint: Starting from a supposed point in $X - f(X)$, apply f repeatedly.)

(b) Give an example of a non-compact metric space Y and a non-surjective isometric function $g \colon Y \to Y$.