MATH 5365 MIDTERM

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Name: ____________________________________

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1. Let $X = \mathbb{R}$ and let $Y = [1, 3) \cup (4, 6]$ with the subspace topology. Let $A = [2, 3) \cup (5, 6]$.

(a) What is the closure of A in X?

(b) What is the interior of A in X?

(c) What is the closure of A in Y?

(d) What is the interior of A in Y?

(No proofs required.)

2. Let $X = \{1, 2, 3\}$ with the topology generated by the base $\mathcal{B}_X = \{\{1, 2\}, \{3\}\}$. Let $Y = \{4, 5, 6, 7\}$ with the topology generated by the base $\mathcal{B}_Y = \{\{4\}, \{5\}, \{5, 6, 7\}\}$. Y has four 3-point subspaces, namely, the sets $\{4, 5, 6\}$, $\{4, 5, 7\}$, $\{4, 6, 7\}$, and $\{5, 6, 7\}$ with the corresponding subspace topologies.

(a) Is there a continuous bijection from X to any of these subspaces? If yes, then to which ones?

(b) Is there a homeomorphism from X to any of these subspaces? If yes, then to which ones?

(No proofs required.)

3. Let (X, d) be a metric space and let $\mathcal{T}$ be the topology induced by d. Let $\mathcal{U}$ be the corresponding product topology on $X \times X$. Prove that d is a continuous function from $(X \times X,\ \mathcal{U})$ to $\mathbb{R}$. You may assume the fact that if $d(y, z) < \varepsilon$, then $|d(x, y) - d(x, z)| < \varepsilon$. (You can also prove this fact for extra credit.)