MATH 5365 Final Exam, May 13, 2015

Name: _____

1. Show the incompleteness of (a, b), $(a, b]$, and $[a, b)$ by exhibiting Cauchy sequences that do not converge.

2. Given K compact, $p \in Y$, and an open $U \subset K \times Y$ such that $K \times \{p\} \subset U$, prove that there exists an open $V \subset Y$ such that $p \in V$ and $K \times V \subset U$.

3. Prove that if M is a connected metric space with at least two points, then M is uncountable.

4. Prove that if X and Y are compact Hausdorff spaces and f is a continuous surjection from X to Y, then f is a quotient map.

5. Prove that if I is an uncountable set, then the product space $\mathbb{R}^I$ is not metrizable.