

Name: \_\_\_\_\_

1. Show the incompleteness of  $(a, b)$ ,  $(a, b]$ , and  $[a, b)$  by exhibiting Cauchy sequences that do not converge.
2. Given  $K$  compact,  $p \in Y$ , and an open  $U \subset K \times Y$  such that  $K \times \{p\} \subset U$ , prove that there exists an open  $V \subset Y$  such that  $p \in V$  and  $K \times V \subset U$ .
3. Prove that if  $M$  is a connected metric space with at least two points, then  $M$  is uncountable.
4. Prove that if  $X$  and  $Y$  are compact Hausdorff spaces and  $f$  is a continuous surjection from  $X$  to  $Y$ , then  $f$  is a quotient map.
5. Let  $(X, d)$  be a metric space with nonempty subset  $A$ . Show that  $f(x) = d(x, A)$  defines a continuous function from  $X$  to  $\mathbb{R}$ .