Exercise 1.10

$$\ell = \prod_{i=0}^{4} p(i)^{n_i} \leftarrow \begin{cases} \text{where } p(y) = e^{-\mu}\mu^y / y! \\ \& \ \vec{n} = (109, 65, 22, 3, 1) = (n_0, \dots, n_4) \end{cases}$$

$$L = \log \ell = \ln \ell$$

$$L = \sum_{i=0}^{4} n_i(-\mu + i \log\mu + \text{constant})$$

solve $0 = dL/d\mu = \sum_{i=0}^{4} n_i(-1 + i/\mu) = \left(\sum_{i=0}^{4} i n_i\right)\frac{1}{\mu} - \sum_{i=0}^{4} n_i.$

solution: $\hat{\mu} = \left(\sum_{i=0}^{4} i n_i\right) \Big/ \left(\sum_{i=0}^{4} n_i\right) = 122/200 = \boxed{0.61}$

($\hat{\mu}$ is the same as the sample mean $\frac{\text{total deaths}}{\text{trials}}$.)

(Recall that μ is the mean of the Poisson distrib.)

For our 5 categories $(0, 1, 2, 3, 4)$, the MLE expected frequences are $\hat{\mu}_0, \dots, \hat{\mu}_4 = np(0), \dots, np(4)$ where $\mu = \hat{\mu}$: 108.7, 66.3, 20.2, 4.1, 0.6

We've only 1 degree of freedom, μ, so

$$\chi_1^2 = z^2 = \sum_{i=0}^{4} \frac{(n_i - \mu_i)^2}{\mu_i} = 0.71 \quad \& \quad z \sim N(0,1).$$

$$P(z^2 \geq 0.71) = P(|z| \geq \underbrace{\sqrt{0.71}}_{0.84}) = 0.40 = 40\% > 5\%.$$

The χ^2 test does not reject the Poisson hypothesis.

To compute P above, use $1 - \operatorname{erf}(.84/\sqrt{2})$ or, in R, 2*(1-pnorm(.84)) or 1-pchisq(.71,1). (R also has sqrt() for finding square roots.) Or do the integral on your calculator: $1 - \left[2\left(\int_0^{.84} e^{-x^2/2}\,dx\right)/\sqrt{2\pi}\right]$.