3D #4 "only if" proof:

Choose a basis $w_1, \ldots, w_m$ of range T_2.
Choose $v_i \in V$ such that $w_i = T_2 v_i$ for each $i \le m$. Let $x_i = T_1 v_i$ for $i \le m$.

Claim. $x_1, \ldots, x_m$ is a basis of range T_1.
Proof.
$$\begin{aligned} \dim \operatorname{range} T_1 &= \dim W - \dim \operatorname{null} T_1 \\ &= \dim W - \dim \operatorname{null} T_2 \\ &= \dim \operatorname{range} T_2 \\ &= m. \end{aligned}$$
So, it's enough to verify independence.
$$0 = a_1 x_1 + \cdots + a_m x_m$$
$$\Rightarrow 0 = a_1 T_1 v_1 + \cdots + a_m T_1 v_m = T_1(a_1 v_1 + \cdots a_m v_m)$$
$$\Rightarrow a_1 v_1 + \cdots + a_m v_m \in \operatorname{null} T_1 = \operatorname{null} T_2$$
$$\Rightarrow 0 = T_2(a_1 v_1 + \cdots a_m v_m) = a_1 w_1 + \cdots + a_m w_m$$
$$\Rightarrow 0 = a_1 = \cdots = a_m$$
because $w_1, \ldots, w_m$ is lin. indep. $\blacksquare$

Extend $w_1, \ldots, w_m$ to a basis $w_1, \ldots, w_n$ of W; extend $x_1, \ldots, x_m$ to a basis $x_1, \ldots, x_n$ of W. Define $S \in \mathcal{L}(W)$ by $S w_i = x_i$. Then S^{-1} exists; it is the map in $\mathcal{L}(W)$ sending x_i to w_i. Given $v \in V$, we next verify that $S T_2 v = T_1 v$. Let $T_2 v = b_1 w_1 + \cdots + b_m w_m$. Then $S T_2 v = b_1 x_1 + \cdots b_m x_m = T_1(b_1 v_1 + \cdots + b_m v_m)$. Therefore, it's enough to prove $T_1 v = T_1 y$ where $y = b_1 v_1 + \cdots b_m v_m$.

$$
\begin{aligned}
T_1 v = T_1 y &\iff T_1(v-y) = 0 \\
&\iff v-y \in \operatorname{null} T_1 \\
&\iff v-y \in \operatorname{null} T_2 \\
&\iff T_2(v-y) = 0 \\
&\iff T_2 v = T_2 y.
\end{aligned}
$$

So, proving $T_2 v = T_2 y$ is enough.

$T_2 y = b_1 T_2 v_1 + \cdots b_m T_2 v_m = b_1 w_1 + \cdots = T_2 v.$ ■