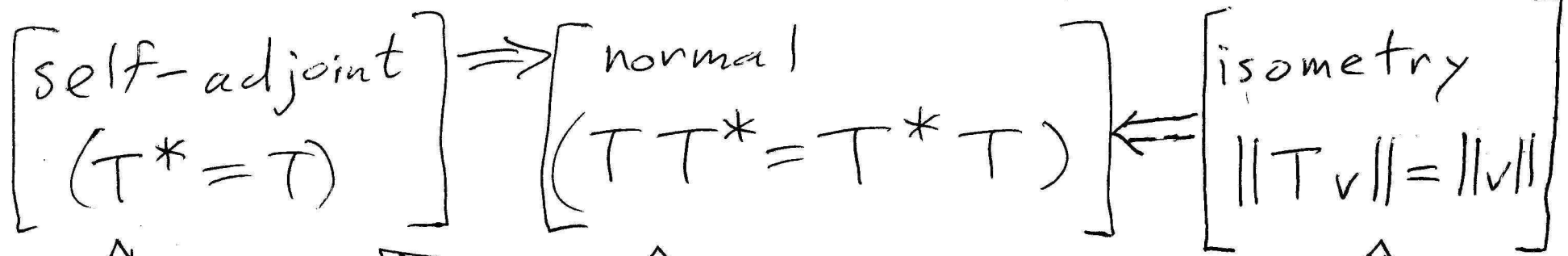


Ch. 7 Summary: $\langle v, T^*w \rangle = \langle Tv, w \rangle$ defines T^* .



$F = \mathbb{R}$ \swarrow $\updownarrow F = \mathbb{C}$

$$M(T) = \begin{pmatrix} \lambda_1 & & & \\ & \ddots & & \\ & & \bigcirc & \\ & & & \ddots \\ & & & & \lambda_n \end{pmatrix}$$

for some orth. e_1, \dots, e_n as input & output basis

\updownarrow $\left[T^* = T^{-1} \right]$

$\updownarrow F = \mathbb{C}$

$$M(T) = \begin{pmatrix} z_1 & & & \\ & \ddots & & \\ & & \bigcirc & \\ & & & \ddots \\ & & & & z_n \end{pmatrix}$$

with $|z_1| = \dots = |z_n| = 1$ for some orth basis e_1, \dots, e_n for input & output

\uparrow $\left[\begin{array}{l} \text{positive} \\ (T^* = T \ \& \ \langle Tv, v \rangle \geq 0) \end{array} \right]$

\updownarrow

$$M(T) = \begin{pmatrix} \lambda_1 & & & \\ & \ddots & & \\ & & \bigcirc & \\ & & & \ddots \\ & & & & \lambda_n \end{pmatrix}$$

& $\lambda_1, \dots, \lambda_n \geq 0$
for some orth. basis e_1, \dots, e_n for input & output

These all assume $T \in \mathcal{L}(V)$ and $\dim V = n < \infty$ and an inner product.

\updownarrow $\left[T \text{ has positive square root} \right]$
 $M(\sqrt{T}) = \begin{pmatrix} \sqrt{\lambda_1} & & & \\ & \ddots & & \\ & & \bigcirc & \\ & & & \ddots \\ & & & & \sqrt{\lambda_n} \end{pmatrix}$ for some orth basis e_1, \dots, e_n for input & output

$$(ST)^* = T^* S^*$$

$$T^{**} = T$$

$$(T^* T)^* = T^* T$$

properties of the
adjoint

$$M(T^*) = \overline{M(T)}^t$$

when using an orthonormal
basis e_1, \dots, e_n for
both input & output

$T^* T$ is always
positive.

Polar Decomposition:

$$\forall T \in \mathcal{L}(V) \exists S \in \mathcal{L}(V)$$

S is an isometry and

$$T = S \sqrt{T^* T}.$$

Singular Value Decomposition:

$$\forall T \in \mathcal{L}(V) \exists (\text{orthonormal bases } e_1, \dots, e_n \text{ \& } f_1, \dots, f_n \text{ of } V)$$

$$\forall j \leq n \quad T e_j = s_j f_j \text{ where}$$

$$s_j = \sqrt{\lambda_j} \quad \& \quad T^* T e_j = \lambda_j e_j.$$

This all assumes $\dim V < \infty$ and
an inner product on V .