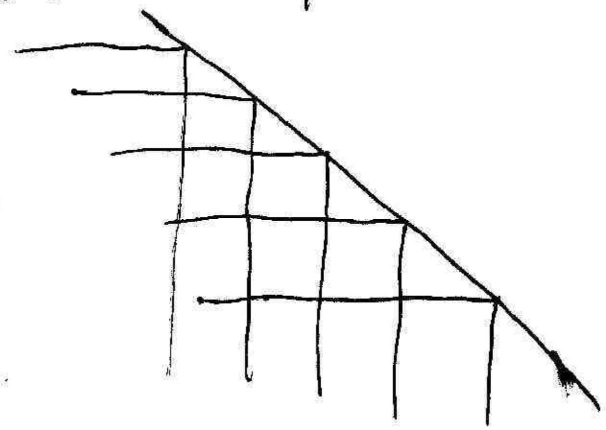
HW1

1. Prove or disprove: $f(X-Y) \subset f(X) - f(Y)$
Repeat for $f(X-Y) \supset f(X) - f(Y)$.

2. (Read section §2 of Munkres if you are not familiar with "injection," "surjection," and "bijection.") Prove that if A & B are not empty, then there are injections $f : A \to A \times B$ and $g : B \to A \times B$

3. Express $\{(x,y) \in \mathbb{R} \times \mathbb{R} \mid x \leq y\}$ as a union of Cartesian products $\bigcup_{z \in \mathbb{R}} (I_z \times J_z)$.

Hint :



$I_z = ?$

$J_z = ?$

4 (Grad. only) Prove that the set in 3 is not a finite union $\bigcup_{i=1}^{n} (A_i \times B_i)$ of Cartesian products of subsets of $\mathbb{R}$.

5 Which of the following sets are functions from $\{1, 2, 3\}$ to $\{1, 2\}$?

$f = \{(2, 3), (2, 2), (1, 2)\}$

$g = \{(3, 2), (1, 2), (2, 1)\}$

$h = \{(1, 1), (2, 2), (1, 3)\}$

$k = \{(1, 1), (1, 1), (2, 2), (3, 1)\}$

6 (Grad. only) Prove that, if $A \neq \emptyset$, then

$$\bigcap_{a \in A} \bigcup_{b \in B} S_{(a,b)} = \bigcup_{f \in B^A} \bigcap_{a \in A} S_{(a, f(a))}$$ where...

... $B^A = \{ f \mid f : A \to B \}$ (and each $S_{(a,b)}$ is a set).

(7) Express $[5, 7)$ as a union $\bigcup_{n \in \mathbb{N}} [a_n, b_n]$ of closed intervals. $a_n = ?$ $b_n = ?$

(8) Prove that $[5, 7)$ is not a finite union $\bigcup_{i=1}^{n} [a_i, b_i]$ of closed intervals.

(9) Give an example of three objects $a, A, \mathcal{A}$ such that $a \in A$ and $A \in \mathcal{A}$ but $a \notin \mathcal{A}$.

(10) A set $S \subset \mathbb{R}$ is <u>convex</u> iff $\forall x \in S \ \forall y \in S \ \forall z \in \mathbb{R} \ (x < z < y \Rightarrow z \in S)$. Give an example of $f : \mathbb{R} \to \mathbb{R}$ and convex $S \subset \mathbb{R}$ such that $f^{-1}(S)$ is not convex.