

① (Grad. only) Prove that \mathbb{N}^2 with the dictionary order has the l.u.b. property. Hint: Every nonempty

$S \subset \mathbb{N}$ (with the usual $<$) has a least element.

② Prove that \mathbb{R}^2 with the dictionary order does not have the l.u.b. property.

Hint: Explain why $\{0\} \times \mathbb{R} = \{(0, x) \mid x \in \mathbb{R}\}$ has upper bounds but no least upper bound.

③ Prove that if E_1 & E_2 are equivalence relations on A , then $E_1 \cap E_2$ is too.

④ Find an isomorphism from the interval $[0, 1)$ with its usual ordering $<$ to...

... the interval $[0, \infty)$ with its usual ordering.

⑤ Express $\left(\prod_{i=1}^4 A_i\right) - \left(\prod_{i=1}^4 B_i\right)$ as a union of four disjoint Cartesian products.