HW8

① Prove that if $X = \prod_{n \in \mathbb{N}} X_n$, $a = (a_1, a_2, a_3, \ldots) \in X$, and A is the set of all $b \in X$ such that, for some $n_b \in \mathbb{N}$, $a_m = b_m$ for all $m \geq n_b$, then $\overline{A} = X$.

② [Grad only] Prove that there is a countable set C such that $\overline{C} = \mathbb{R}^\omega$.

③ The coordinate projections $\pi_\beta : \prod_{\alpha \in J} X_\alpha \to X_\beta$ are defined by $\pi_\beta(p) = p(\beta)$ for all $\beta \in J$ and all $p \in \prod_{\alpha \in J} X_\alpha$. Prove that a topology $\mathcal{T}$ on X $\left(X = \prod_{\alpha \in J} X_\alpha\right)$ is finer than the product topology if and only if $\mathcal{T}$ makes every π_β continuous.

HW 8

(4) Recall from HW1, the problem of showing there exist injections $X \to X \times Y$ & $Y \to X \times Y$, assuming $X, Y \neq \emptyset$. Now, assuming J is a set and X_α is a nonempty space for each $\alpha \in J$, prove that, for each $\alpha \in J$, there is an embedding from X_α to $X = \prod_{\alpha \in J} X_\alpha$.

(5) Prove that $\mathbb{N} \cong \mathbb{Z} \not\cong \mathbb{Q}$, where this notation means that there is a homeomorphism from $\mathbb{N}$ to $\mathbb{Z}$ but there is none from $\mathbb{Z}$ to $\mathbb{Q}$. ($\mathbb{N}, \mathbb{Z}, \mathbb{Q}$ are subspaces of $\mathbb{R}$.)