

HW
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- ① Prove that $\mathbb{R}^2 - \mathbb{Q}^2$ is connected.
- ② Prove that $(\mathbb{R} - \mathbb{Q})^2$ is disconnected.
- ③ How many non-homeomorphic connected spaces with exactly three points exist? That is, how long is the longest list of 3-point connected spaces X_1, \dots, X_n such that no homeomorphism $h: X_i \rightarrow X_j$ exists for any pair $i \neq j$?

④ $X = [0, 1] / \sim$ with the quotient topology where $x \sim y \iff (x = y \text{ or } x, y \in \{0, 1\})$.

$Y = [0, 2] / \equiv$ with the quotient topology where $x \equiv y \iff (x = y \text{ or } x, y \in \{0, 1, 2\})$.

Which of X & Y is a circle (\circ)?

Which of X & Y is a figure eight (∞)?

Prove that X & Y are not homeomorphic.