HW12

(1) [Grad only] In a space X, let $p \sim q$ if there is a connected $Y \subset X$ such that $p, q \in Y$. The $\sim$-equivalence classes are called components. (See §25.) Let $X = \{0\} \cup \bigcup_{n \in \mathbb{N}} \left[\frac{1}{2n}, \frac{1}{2n-1}\right)$. Find $A \subset \mathbb{R}$ such that $A \cong X/\sim$ (where $X/\sim$ has the quotient topology and $Z \cong W$ means there exists a homeomorphism $h: Z \to W$).

(2) Prove that if X & Y are path connected, then so is $X \times Y$. Hint: Look at how Munkres proves Thm. 23.6.

(3) [Grad only] Prove that if a connected metric space has two or more points, then it is uncountable. You may assume the claim of Exercise 20.3(a).