

① [Grad only] In a space X , let $p \sim q$ HW12
if there is a connected $Y \subset X$ such that $p, q \in Y$.
The \sim -equivalence classes are called components.

(See §25.) Let $X = \{0\} \cup \bigcup_{n \in \mathbb{N}} \left[\frac{1}{2n}, \frac{1}{2n-1} \right)$.

Find $A \subset \mathbb{R}$ such that $A \cong X/\sim$ (where X/\sim
has the quotient topology and $Z \cong W$ means
there exists a homeomorphism $h: Z \rightarrow W$).

② Prove that if X & Y are path connected,
then so is $X \times Y$. Hint: Look at how
Munkres proves Thm. 23.6.

③ [Grad only] Prove that if a connected metric
space has two or more points, then it is uncountable.
You may assume the claim of Exercise 20.3(a).