HW 14

(1) Prove that if X is compact, Y is Hausdorff, and $f: X \to Y$ is continuous injection, then f is an embedding.

(2) Repeat (1) with "injection" replaced by "surjection" and "embedding" replaced by "quotient map."

(3) Assume the claim of Exercise 20.3(a). Prove that if X is a metric space and A & B are disjoint compact subspaces of X, then there exists $\delta > 0$ such that $d(a,b) \geq \delta$ for all $(a,b) \in A \times B$.

(4) Give an example of disjoint closed $A, B \subset \mathbb{R}$ that, for each $\delta > 0$, there exists $(a_\delta, b_\delta) \in A \times B$ such that $|a_\delta - b_\delta| < \delta$.