# MATH 4360 MIDTERM II 

INSTRUCTOR: DAVID MILOVICH

Name:

| Exercise | Point Possible | Score |
| ---: | ---: | :--- |
| 1 | 42 |  |
| 2 | 15 |  |
| 3 | 28 |  |
| 4 | 15 |  |
| Total | 100 |  |

1. [42 points] Fill in each blank with "A," "B," "C," or "D" as appropriate.

Let $X_{1}=\mathbb{R}$ with the standard topology. $[2,3)$ is $\qquad$ in $X_{1}$.
A) open, but not closed B) closed, but not open C) open and closed D) neither open nor closed

Let $X_{2}=\mathbb{R}$ with the discrete topology. $[2,3)$ is $\qquad$ in $X_{2}$.
A) open, but not closed B) closed, but not open C) open and closed D) neither open nor closed

Let $X_{3}=[2,4]$ with the subspace topology from $X_{1} .[2,3)$ is $\qquad$ in $X_{3}$.
A) open, but not closed $\quad$ B) closed, but not open $\quad$ C) open and closed $\quad$ D) neither open nor closed

Let $X_{4}=[1,3)$ with the subspace topology from $X_{1} .[2,3)$ is $\qquad$ in $X_{4}$.
A) open, but not closed B) closed, but not open C) open and closed $\quad$ D) neither open nor closed

Let $X_{5}=[1,3) \cup[4,5]$ with the subspace topology from $X_{1}$. [2,3) is $\qquad$ in $X_{5}$.
A) open, but not closed B) closed, but not open C) open and closed $\quad$ D) neither open nor closed

Let $X_{6}=[1,3) \cup[4,5]$ with the order topology. [2,3) is $\qquad$ in $X_{6}$.
A) open, but not closed B) closed, but not open C) open and closed D) neither open nor closed
2. [15 points] Let $X=\{1,2\}$ with the topology $\mathcal{T}_{X}=\{\varnothing,\{1\},\{1,2\}\}$. Let $Y=\{3,4\}$ with the topology $\mathcal{T}_{Y}=\{\varnothing,\{3\},\{4\},\{3,4\}\}$. Give an example of a basis $\mathcal{B}$ for the product topology on $X \times Y$. Explicitly list the elements of your example $\mathcal{B}$.
3. [28 points] Let $X=\{1,2,3\} \times\left(\{0\} \cup \mathbb{Z}_{+}\right)$.
(a) Prove that $X$ is countable.
(b) Prove that $X$ with the dictionary order is well-ordered.
(You may assume that $\mathbb{Z}_{+}$is countable and well-ordered.)
4. [15 points] Give an example of a 6 -element topology $\mathcal{T}=\left\{U_{1}, U_{2}, U_{3}, U_{4}, U_{5}, U_{6}\right\}$ on the set $X=\{1,2,3\}$. Explicitly list the elements of your example $\mathcal{T}$.

