If A is ______, then $\exists f$ ...	countable (ctbl.)			
	empty	finite, nonempty	countably infinite	uncountable
$\exists f: A \to \mathbb{N}$ injective	yes	yes	yes	no
$\exists f: A \to \mathbb{N}$ surjective	no	no	yes	yes
$\exists f: A \to \mathbb{N}$ bijective	no	no	yes	no
$\exists f: \mathbb{N} \to A$ injective	no	no	yes	yes
$\exists f: \mathbb{N} \to A$ surjective	no	yes	yes	no
$\exists f: \mathbb{N} \to A$ bijective	no	no	yes	no
	finite		infinite	

$(\forall n \in \mathbb{N} \quad A_n \text{ countable}) \Rightarrow \bigcup_{n \in \mathbb{N}} A_n \text{ countable}$

$B_1, B_2, B_3, \ldots, B_n$ ctbl. $\Rightarrow B_1 \times B_2 \times \cdots \times B_n$ ctbl.

$\& \; B_1 \cup \cdots \cup B_n$ ctbl.

C ctbl. $\&$ $D \subset C \Rightarrow D$ ctbl.

E unctbl. $\&$ F ctbl. $\Rightarrow E - F$ unctbl.

$\hookrightarrow$ (Proof: $E-F$ $\&$ F ctbl. $\Rightarrow E \subset (E-F) \cup F$ ctbl. $\Rightarrow E$ ctbl.)

Warning

- Do not confuse different "levels" of sets:

$\mathcal{A} = \{[x, x+1) \mid x \in \mathbb{Q}\}$ is a countable set of uncountable sets.

$\mathcal{B} = \{\{\lceil nx \rceil \mid n \in \mathbb{N}\} \mid x \in \mathbb{R}\}$ is an uncountable set of countable sets.

- $\bigcup \mathcal{A}$ is a countable union of unctbl. sets;
  $\bigcup \mathcal{A}$ is an uncountable set: $\bigcup \mathcal{A} = \mathbb{R}$.
- $\bigcup \mathcal{B}$ is an uncountable union of countable sets.
  But $\bigcup \mathcal{B}$ is a countable set: $\bigcup \mathcal{B} = \mathbb{Z}$.