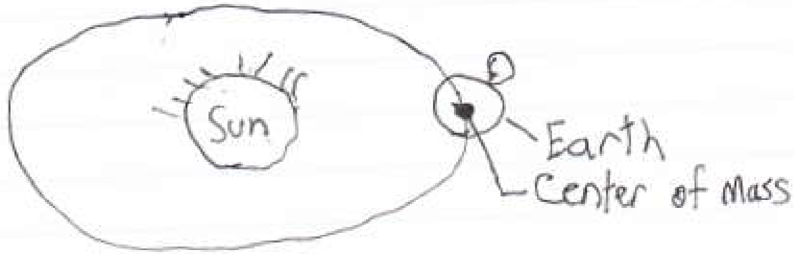
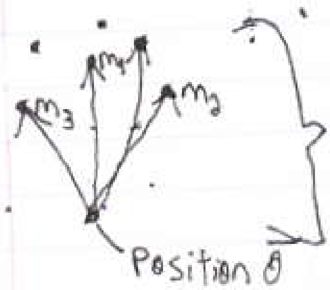
3-10-2010

First: Center of Mass
- Every rigid object's motion breaks into 2 parts:
  1. Translational motion of its CM.
  2. Rotation (Spinning about its CM.



$\vec{r}_{cm} = \frac{1}{M}\sum \vec{r}_i m_i$



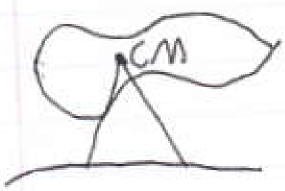
Center of mass of this collection of 8 point masses

$M = \sum m_i$

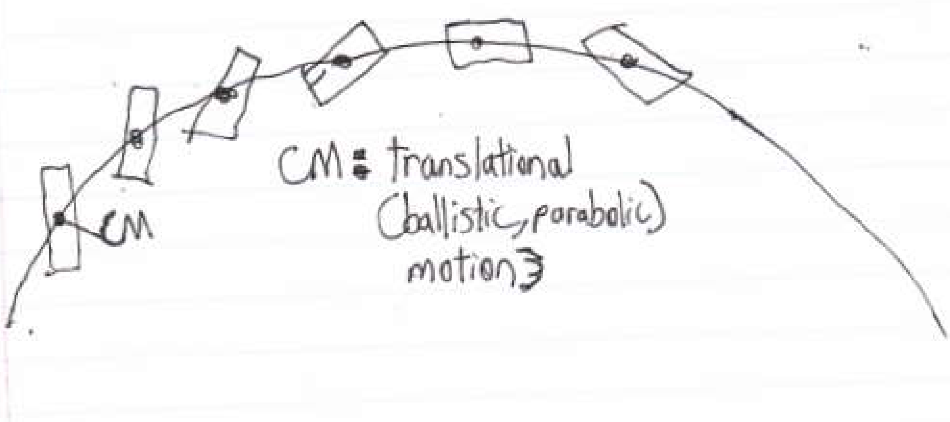
$\vec{r}_{cm} = \frac{1}{M}\int \vec{r}\, dm$

$M = \int dm$

Thin, flat plate



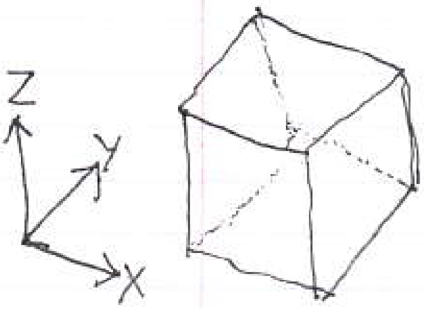
The CM is the only place you can balance this plate on the cone.



CM: translational (ballistic, parabolic) motion

2

An empty box with an open top and thin faces weighs 1 kg and has dimensions 50cm x 50cm x 50cm



4 sides and the bottom, no top.

$\boxed{\text{Find the center of mass of this box assuming uniform density}}$

$\vec{r}_{cm} = \frac{1}{M}\sum \vec{r}_i m_i$

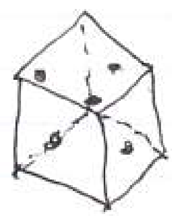
$x_{cm} = \frac{1}{M}\sum x_i m_i$

$y_{cm} = \frac{1}{M}\sum y_i m_i$

$z_{cm} = \frac{1}{M}\sum z_i m_i$

Each side has mass $\frac{M}{5}$ and $z_{cm} = 25\,cm$

The bottom has mass $\frac{M}{5}$ and $z_{cm} = 0\,cm$

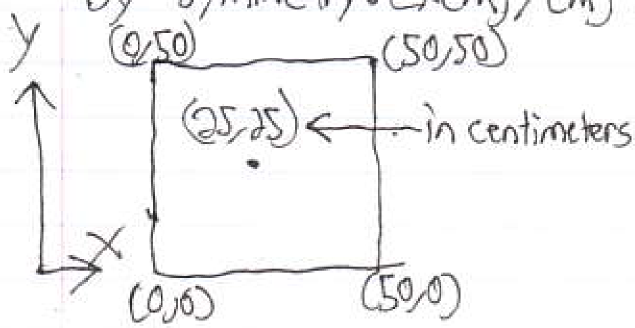


Replace each ~~mass~~ face with a point mass at its CM

$$\underset{\text{whole Box}}{z_{cm}} = \frac{1}{M}\Big(\underbrace{25cm \cdot \frac{M}{5} \cdot 4}_{\text{4 sides}} + \underset{\text{bottom}}{0}\Big)$$

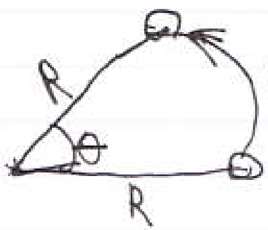
$z_{cm} = 20\,cm$

By symmetry: (x_{cm}, y_{cm}) is in "center" of box



Look at box from above

Spinning



ℓ = arc length

$\theta = \frac{\ell}{R}$ (in radians)

2π radians $= 360° = 1$ revolution

$\Delta\theta$ small $\Rightarrow |\Delta r| \approx \ell = R\Delta\theta$

$\therefore \Rightarrow \left|\frac{\Delta r}{\Delta t}\right| \approx R\left|\frac{\Delta\theta}{\Delta t}\right|$

$\Delta\theta = \frac{\ell}{R} \Rightarrow \ell = R\Delta\theta$

$\bar{v}$

$\bar{v}$ = avg speed equals R times avg. angular speed

$|\vec{r}_1| = |\vec{r}_2| = R$

$\Delta\vec{r}$ = (linear) displacement

$\Delta\theta$ = angular displacement

$\frac{\Delta\theta}{\Delta t}$ = average angular velocity

(Choose + and − rotation directions)

ω = instantaneous angular speed: $\left|\frac{d\theta}{dt}\right|$

$\bar{\omega}$ is average angular velocity

$\omega = \left|\frac{d\theta}{dt}\right| \qquad v = \left|\frac{d\vec{r}}{dt}\right|$

$v = R\omega$

~~$a = R\alpha$~~

$a = R\alpha$

$a = |\vec{a}| \qquad \vec{a} = \frac{d\vec{v}}{dt}$

~~$\alpha = d\omega$~~

$\alpha = \left|\frac{d\omega}{dt}\right|$ = magnitude of angular acceleration

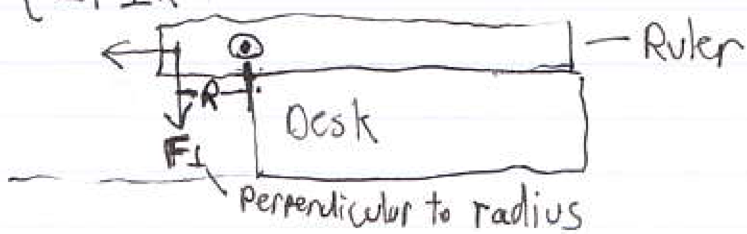
$\vec{F} = m\vec{a}$

Angular version of ↑

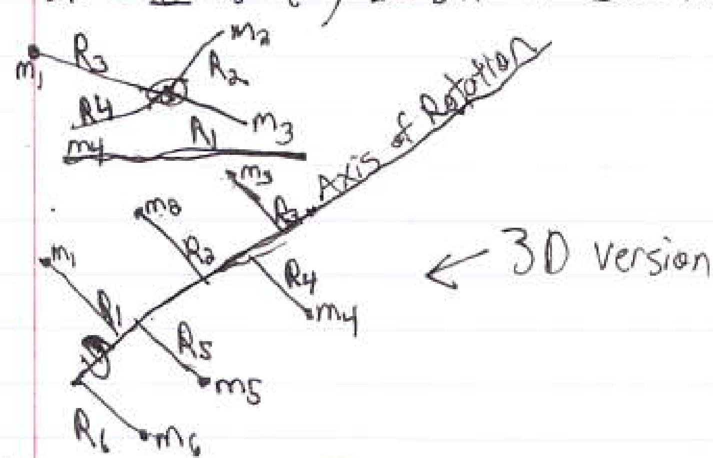
$\tau = I\alpha$

Torque — moment of inertia — angular acceleration ~~angluar~~

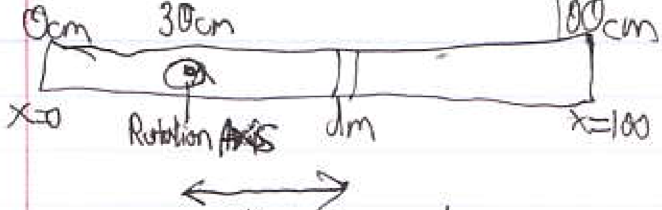
$\tau = F_{\perp} R$



$I = \sum R_i^2 m_i$; $I = \int R^2 dm$ (Continuous Version)



Moment of Inertia depends on the axis of rotation used.



$R = |x-30|$

$$I = \int_{ruler} R^2\, dm = \int_0^{100cm} (x-30)^2\, dm$$

$dm = \rho\, dx$ where $\rho = \frac{\text{length of ruler}}{\text{mass of ruler}}$

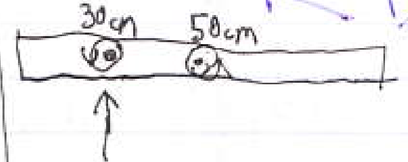
$$\cancel{I = \int_0^{100cm} (x-30)^2}$$

$$I = \int_0^{100cm} (x-30)^2 \rho\, dx$$

Lots of integrals on page 260

From page 260...

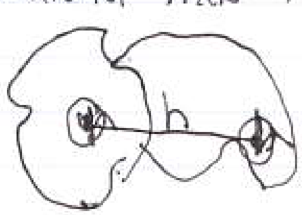
$\therefore I_{CM} = \frac{1}{2} M (100cm)^2$ 1/12



$$I = I_{cm} + M(20cm)^2$$
$$= \frac{1}{2} M(100cm)^2 + M(20cm)^2$$

$\frac{1}{12}$

Parallel Axis Theorem



mass:	m_1	m_2
moments of inertia:	I_1	I_2
Parallel Axis of Rotation:	A_1	A_2

I = moment of inertia of the whole blob about axis A_2.

Not simply $I_1 + I_2$
Instead: (1) Find CM of whole blob. (2) Find I_{CM}.

(3) $I = I_{CM} + M\ell^2$

ℓ distance from CM to axis A_2

Linear Angular

$\vec{v} = \frac{d\vec{r}}{dt}$

$v = \left|\frac{d\vec{r}}{dt}\right|$ $\omega = \left|\frac{d\theta}{dt}\right|$

$\vec{a} = \frac{d\vec{v}}{dt}$

$a = \left|\frac{d\hat{v}}{dt}\right|$ $\alpha = \frac{d\omega}{dt}$

$\vec{F} = m\vec{a}$ $\tau = I\alpha$

$\cancel{K_{tran} = \frac{1}{2} M v_{cm}^2 \quad \frac{1}{12} \quad K_{rot} = \frac{1}{2} I\omega^2}$ $K_{tran} = \frac{1}{2} M v_{cM}^2$

Total kinetic energy is $K_{rot} = \frac{1}{2} I\omega^2$

$K = \frac{1}{2}(Mv_{cm}^2 + I\omega^2)$

You can also relate v_{cm} and ω

Rolling without slipping $v_{cm} = R\omega$