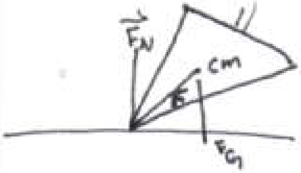
(11.7)

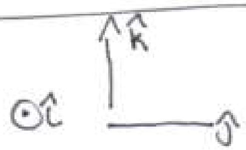
How long does it take for the top's bottom tip to go around this circle

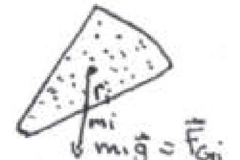
(I ignore friction for simplicity)

$\Sigma \vec{F}_{ext} = \vec{F}_N + \vec{F}_G = \vec{0}$

when the top is not falling quickly really, vertical acc.







$m_i\vec{g} = \vec{F}_{G_i}$

$\vec{g} = (9.8\,m/s^2)(-\hat{k})$

compute the net torque about the cm.

For computing torque draw $\vec{F}_G$ from the CM

Normal force $-\vec{\tau}_N = \vec{r} \times \vec{F}_N$

$$\tau_G = \Sigma \vec{r}_i \times \vec{F}_{G_i} = \Sigma \vec{r}_i \times m_i\vec{g}$$

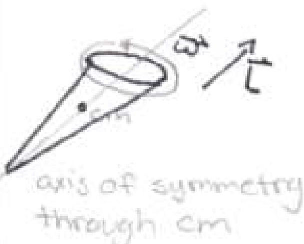
$$\tau_G = (\Sigma m_i \vec{r}_i) \times \vec{g} = M\underbrace{\vec{r}_{cm}}_{\vec{0}} \times \vec{g} = \vec{0}$$

$-\vec{\tau}_G = \vec{0} \times \vec{F}_G = \vec{0}$

$$\vec{r}_{cm} = \frac{\Sigma m_i \vec{r}_i}{m}$$

$m = \Sigma m_i$

an arrow from the cm to the cm has length 0.



$$\frac{d\vec{L}}{dt} = \Sigma \vec{\tau}_{ext} = \vec{\tau}_N + \vec{\tau}_G = \vec{\tau}_N + \vec{0} = \underbrace{\vec{r} \times \vec{F}_N}$$

direction ⊗ perpendicular to $\vec{L}$

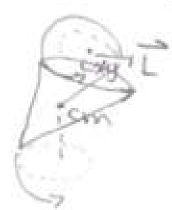
$$\frac{d\vec{L}}{dt} \perp \vec{L}$$

Uniform circular motion

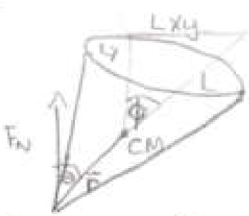
$$\vec{r} \perp \frac{d\vec{r}}{dt}$$

$\Rightarrow \vec{L} = I\vec{\omega}$

moment of inertia from this axis



$$T = \frac{\text{length}}{\text{speed}} = \frac{2\pi r}{v} = \frac{2\pi r}{dr/dt}$$



$L_{xy} = L\sin\phi$

precession period is $T = \dfrac{2\pi L_{xy}}{dL_{xy}/dt}$

$$\frac{d\vec{L}_{xy}}{dt} = \vec{\tau}_{N\,xy} = \vec{\tau}_N$$

$\vec{\tau}_N = \vec{r} \times \vec{F}_N$

$$T = \frac{2\pi L\sin\phi}{\tau_N} = \frac{2\pi I\omega \sin\phi}{rMg\sin\phi}$$

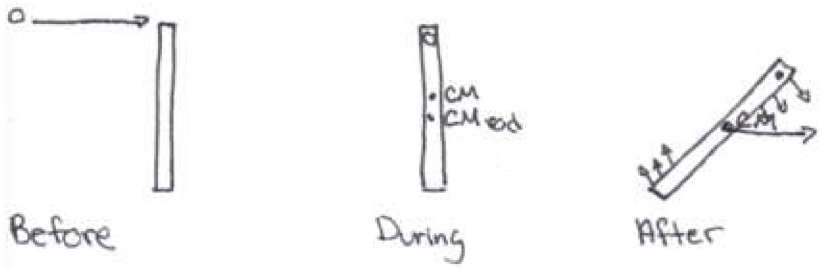
$\tau_N = r\underbrace{F_N}\sin\phi = rMg\sin\phi$

$$\boxed{T = \frac{2\pi I\omega}{rMg}}$$

$F_N \approx F_G = Mg$

$$\frac{dL}{dt} = \tau_N = rMg\sin\phi$$

A thin rod with mass 1.0 kg and length 2.6 m at rest in outer space. (Ignore all external forces.) A tiny, 75 gram rock hits a tip of the rod at a relevative speed of 15 m/s and sticks to the tip of the rod.



Conserve $\vec{P}$ & $\vec{L}$

$$\frac{d\vec{P}}{dt} = \Sigma \vec{F}_{ext} = \vec{0}$$

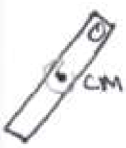
$$\frac{d\vec{L}}{dt} = \Sigma \vec{\tau}_{ext} = \Sigma \vec{r} \times \vec{F}_{ext} = \vec{0}$$

$P \doteq (75\text{ grams}) \cdot 15\text{ m/s} + (1.0\text{kg})(0\text{m/s})$

---

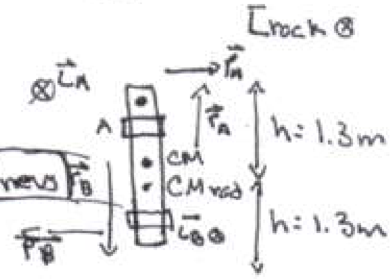
total mass $M = 1.075$ kg $\qquad \vec{P} = M\vec{V}_{cm}$

$$V_{cm} = \frac{P}{M} = \frac{0.075\text{kg} \cdot 15\text{m/s}}{1.075\text{ kg}} =$$



what is ω after the collision? $\boxed{0.13\text{ rev/s}}$

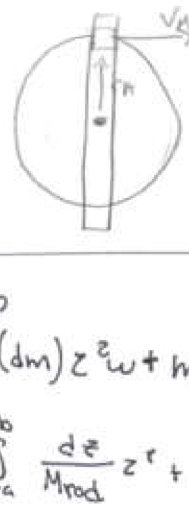
To find out, compute $\vec{\tau}$ at the moment of collision



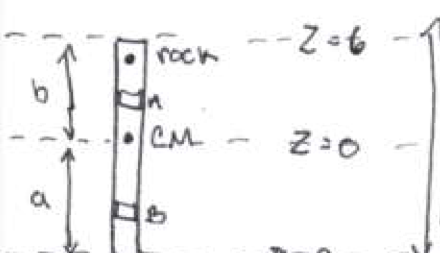
$\vec{L}_A = \vec{r} \times \vec{P}_A$ $\quad \vec{r}_A \quad \vec{P}_A \quad \otimes \vec{L}_A$

$L_A = r_A P_A$ because $\vec{r}_A \perp \vec{P}_A$

$L_A = r_A M_A V_A$



$\omega = \frac{V_A}{r_A} \quad = V_A = r_A \omega$



$L_A = m_A Z_A^2 \omega$

2.6m

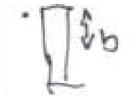
$L_B = M_B Z_B^2 \omega$

$$L = \int_{-a}^{b} (dm) z^2 \omega + m_{rock} b^2 \omega$$

$$L = \omega \left(\int_{-a}^{b} \frac{dz}{M_{rod}} z^2 + M_{rock} b^2 \right)$$

righ before the collision

$L = L_{rock} + 0 \approx b \cdot P_{rock}$



$$b \cdot P_{rock} \approx L = \omega \left(\frac{z^3}{3M_{rod}} \right) \Bigg|_{-a}^{b} + M_{rock} b^2$$

$$\omega = \frac{b \cdot P_{rock}}{\frac{b^3}{3M_{rod}} + \frac{a^3}{3M_{rod}} + M_{rock} b^2} = 0.13\text{ rev/s}$$