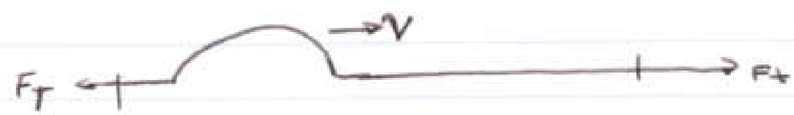
Ch. 15

Homework Due tuesday, April 20, 5 pm

ch. 15 #9, 10 #12, #18, #22, #24, #30, #50, #52, # 56

from last time: speed of trasverse waves on a string



Tension force

string has ~~length~~ L & mass m.
μ = Linear density = $\frac{m}{L}$

$$V = \sqrt{\frac{F_T}{\mu}}$$

$$[V] = \frac{[L]}{[T]}$$

$$[F_T] = \frac{[M][L]}{[T]^2}$$

$$[\mu] = \frac{[M]}{[L]}$$

$$\sqrt{\frac{[F_T]}{[\mu]}} = \sqrt{\frac{[L]^2}{[T]^2}} = \frac{[L]}{[T]}$$

So, Far, we've been doing math model of travelling sine waves:

displacement = $A \sin(kx - \omega t + \phi$

↑ all points of the string have the same amplitude

$V = \frac{\omega}{k}$ (To the right)

What if you add two waves together?

Example: $A\sin(kx - \omega t + \phi_1) + A\sin(kx + \omega t + \phi_2)$

$= 2A\sin\left(\frac{kx - \omega t + \phi + kx + \omega t + \phi_2}{2}\right)\cos\left(\frac{kx - \omega t + \phi_1 - (kx + \omega t + \phi_2)}{2}\right)$

$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$

$\cos(-\theta) = \cos\theta$

$\sin\theta = \cos\left(\theta - \frac{\pi}{2}\right)$ $\sin\left(\theta + \frac{\pi}{2}\right) = \cos\theta$

$= 2A\sin\left(kx + \frac{\phi_1 + \phi_2}{2}\right)\cos\left(-\omega t + \frac{\phi_1 - \phi_2}{2}\right)$

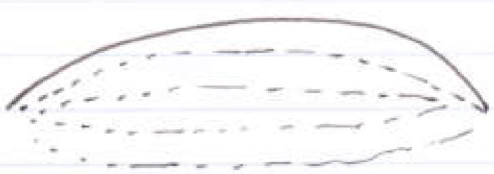
$\cos\left(\omega t + \frac{\phi_2 - \phi_1}{2}\right)$

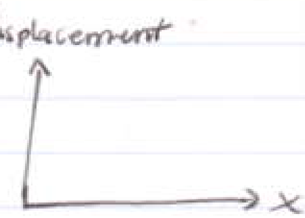
displacement

$2A\sin\left(kx + \frac{\phi_1 + \phi_2}{2}\right)$

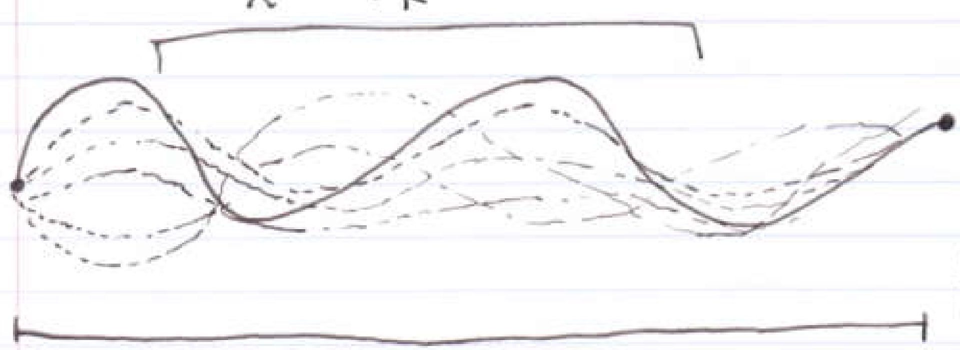
Amplitude is different parts of the string

$\cdot \sin\left(\omega t + \frac{\phi_2 - \phi_1 + \pi}{2}\right)$





$\lambda = 2\pi / k$



$T = \frac{2\pi}{\omega}$

All parts of string with same period

$4\lambda/2 = 2\lambda$

In general, the standing waves of a string of Length L between two end points fixed, satisfy

$$L = \frac{n\lambda}{2} \quad \text{For some } n = 1, 2, 3, \ldots$$

$$\Updownarrow$$

$$\lambda = \frac{2L}{n}$$

$$\frac{w}{k} = v = \sqrt{\frac{F_t}{m}}$$

$$\frac{\lambda w}{2\pi} \Rightarrow w = \frac{2\pi}{\lambda}\sqrt{\frac{F_t}{M}}$$

$$\Rightarrow f = \frac{w}{2\pi} = \frac{1}{\lambda}\sqrt{\frac{F_t}{M}}$$

$$\Rightarrow f = \frac{n}{2L}\sqrt{\frac{F_t}{m}}$$

Tune: Fundament frenquency: $n = 1$

over tone:
- 1st harmonic: $n = 2$
- 2nd harmonic: $n = 3$
- 3rd harmonic: $n = 4$

$$\text{Middle C: } 261.6 \text{ Hz} = \frac{261.6}{s} = \frac{1}{2L}\sqrt{\frac{F_T}{M}}$$