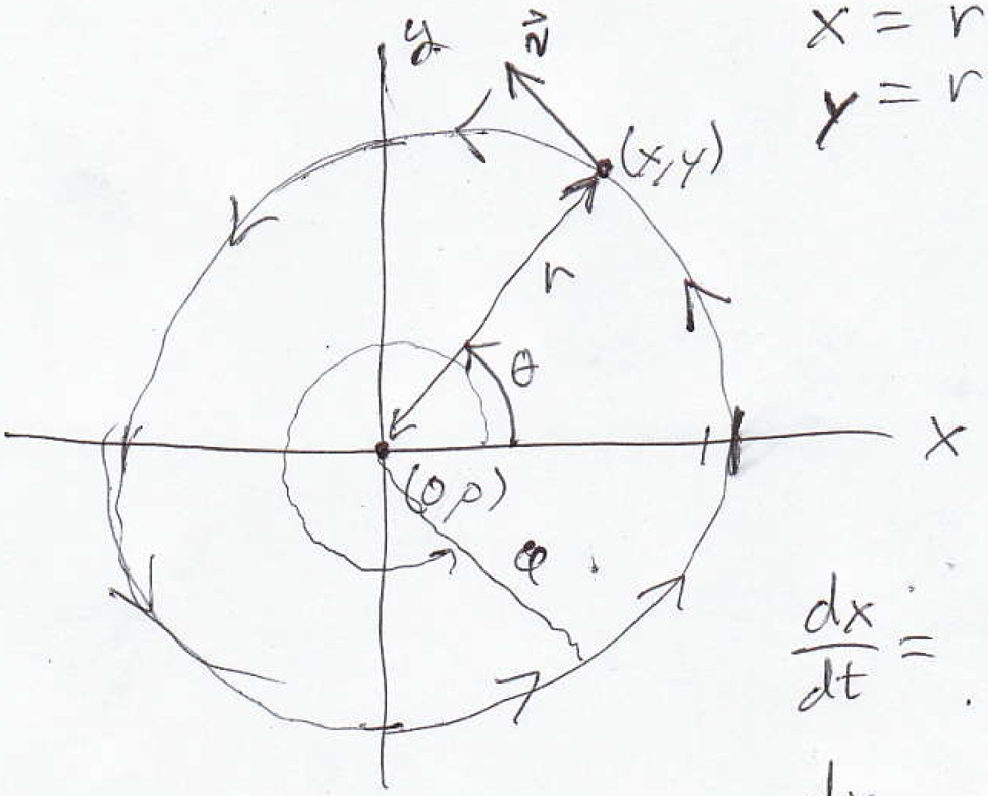
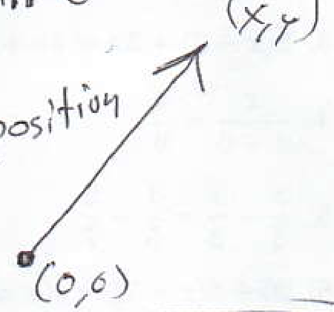
Circular motion (Ch. 5)

(summary p. 130)



$x = r\cos\theta$

$y = r\sin\theta$



$dr/dt = 0$

circular

$$\frac{dx}{dt} = r\frac{d}{dt}\cos\theta$$

$$v_x = \frac{dx}{dt} = r(-\sin\theta)\underbrace{\frac{d\theta}{dt}}_{\omega}$$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{r^2\omega^2(\sin^2\theta + \cos^2\theta)}$$

angular velocity

$$v_y = dy/dt = r(\cos\theta)\underbrace{d\theta/dt}_{\omega}$$

$v = r\omega$

E.g. $r = 0.53\ m$ & $\omega = \underbrace{8\ rev/min}_{\underbrace{8(2\pi)/min}_{8(360^\circ)/min}}$

$v = 27\ m/min$

Non-circular motion:

$$x = r\cos\theta$$

$$v_x = \frac{dx}{dt} = \frac{dr}{dt}\cos\theta + r(-\sin\theta)\frac{d\theta}{dt}$$

Back to circular motion:

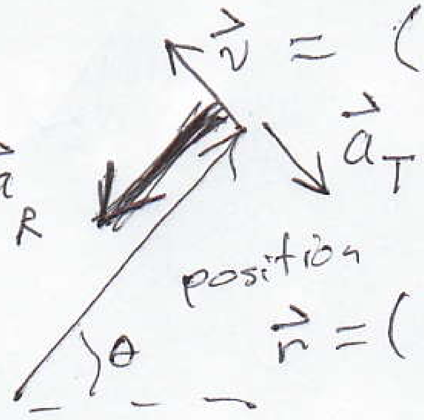
$$v_x = -r\omega\sin\theta \qquad \omega = \frac{d\theta}{dt} \qquad \alpha = \frac{d\omega}{dt}$$

$$v_y = r\omega\cos\theta$$

$$a_x = \frac{dv_x}{dt} = -r\alpha\sin\theta - r\omega(\cos\theta)\omega$$

$$a_y = \frac{dv_y}{dt} = r\alpha\cos\theta + r\omega(-\sin\theta)\omega$$

$$\vec{v} = (-r\omega\sin\theta,\ r\omega\cos\theta)$$



position $\vec{r} = (r\cos\theta, r\sin\theta)$

$$a_T = r\alpha$$

$$\vec{a}_T = r\alpha(-\sin\theta, \cos\theta)$$

$$\vec{a} = \vec{a}_T + \vec{a}_R$$

tangental acceleration

radial accel.

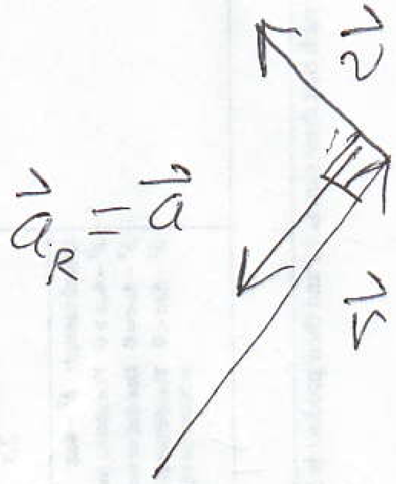
$$\vec{a}_R = -r\omega^2(\cos\theta, \sin\theta)$$

also called centripetal accel.

✓ constant v & ω

$\boxed{\text{uniform circular motion}}$: $\frac{d\omega}{dt} = \alpha \doteq 0$

$$\frac{dv}{dt} = \frac{d}{dt}(r\omega) = r\alpha = 0$$



$a_T = r\alpha = 0$

$a_R = r\omega^2$

$v = r\omega$

$v/r = \omega$

$a = a_R = r\left(\frac{v}{r}\right)^2$

$$a = \frac{v^2}{r}$$

How fast are you spinning around the earth's center? (For simplicity, assume you're living on the equator.)

Use $v = r\omega$.

How much are you accelerating from this spin? Use $a = \frac{v^2}{r}$.

$r = 6.38 \times 10^6\,m$ $\quad \omega = 1\ rev/day$

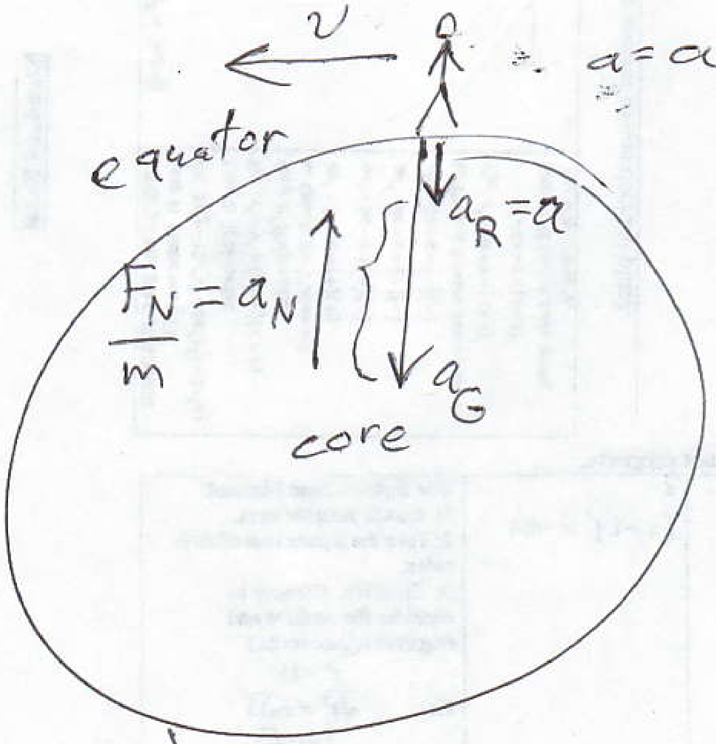
$\omega = 2\pi / (24 \cdot 60 \cdot 60\,s)$

$\omega = 7.27 \times 10^{-5} /s$

$\omega = 7.27 \times 10^{-5}\ rad/s$

$\omega = 4.17 \times 10^{-3}\ deg/s$

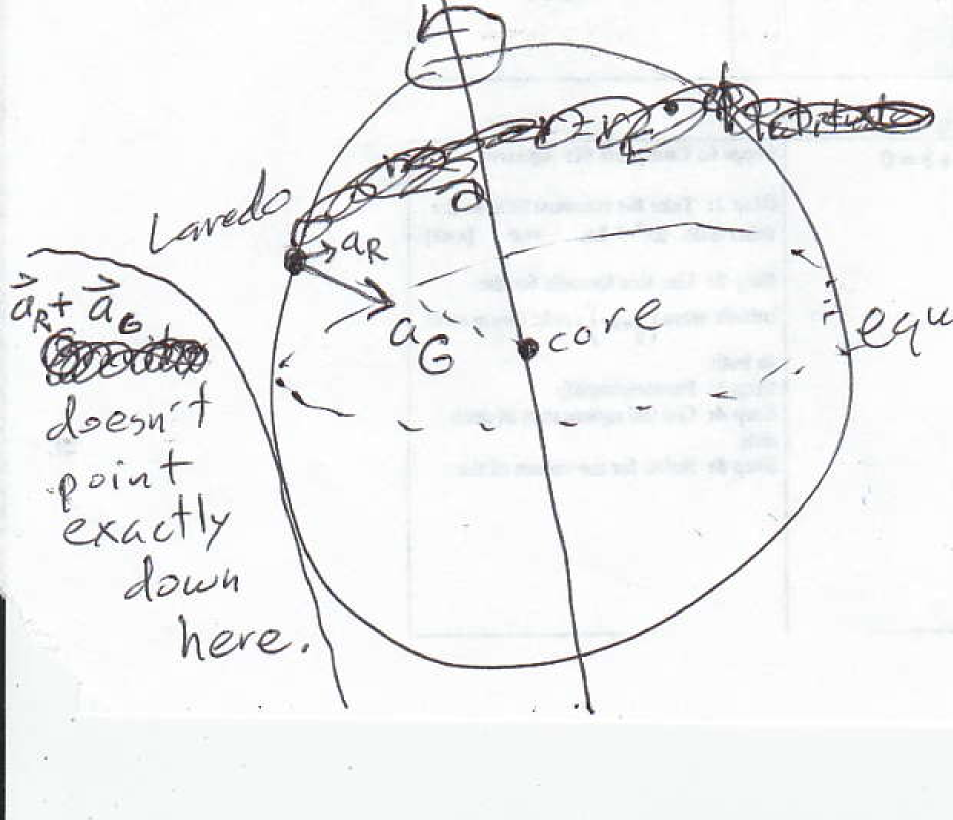
$v = r\omega = 4.64 \times 10^2\ m/s = 1.04 \times 10^3\ mph$

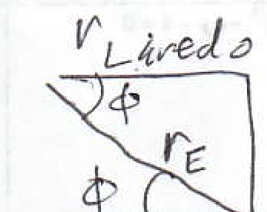


$a = a_R = \dfrac{v^2}{r} = \underbrace{3.37 \times 10^{-2}}_{0.0337}\ m/s^2$

$a_G = 9.80\ m/s^2$

g is perceived as $9.80\ m/s^2$ at the north pole but as $9.77\ m/s^2$ at the equator.





$r_{Laredo} = r_E \cos\phi$

ϕ = latitude

$r_E = 6.38 \times 10^6\,m$