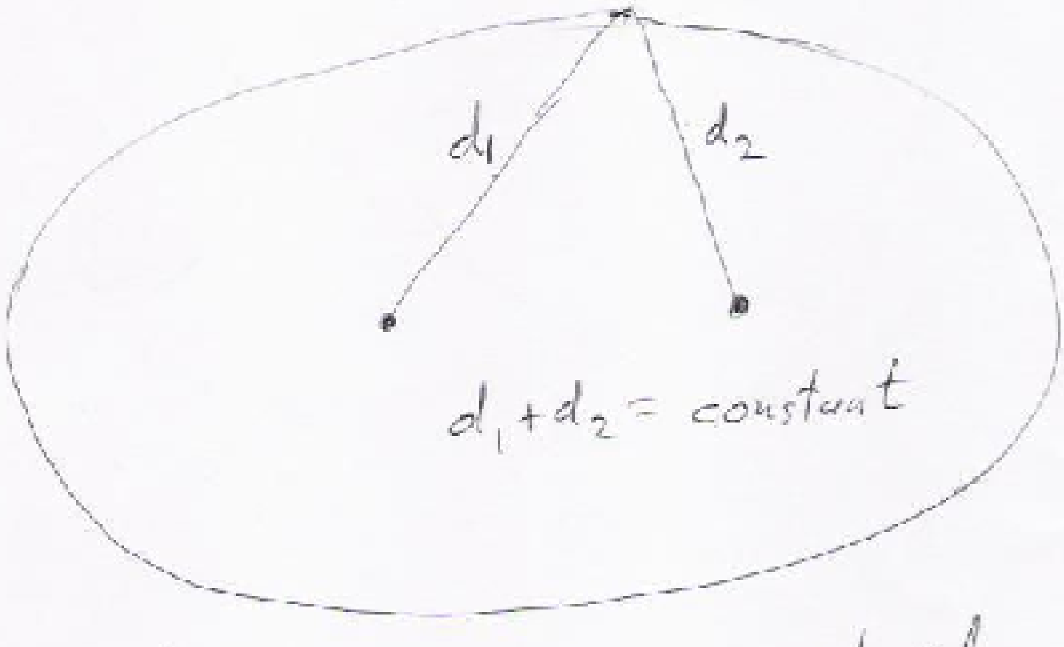
Ch. 6 Gravity

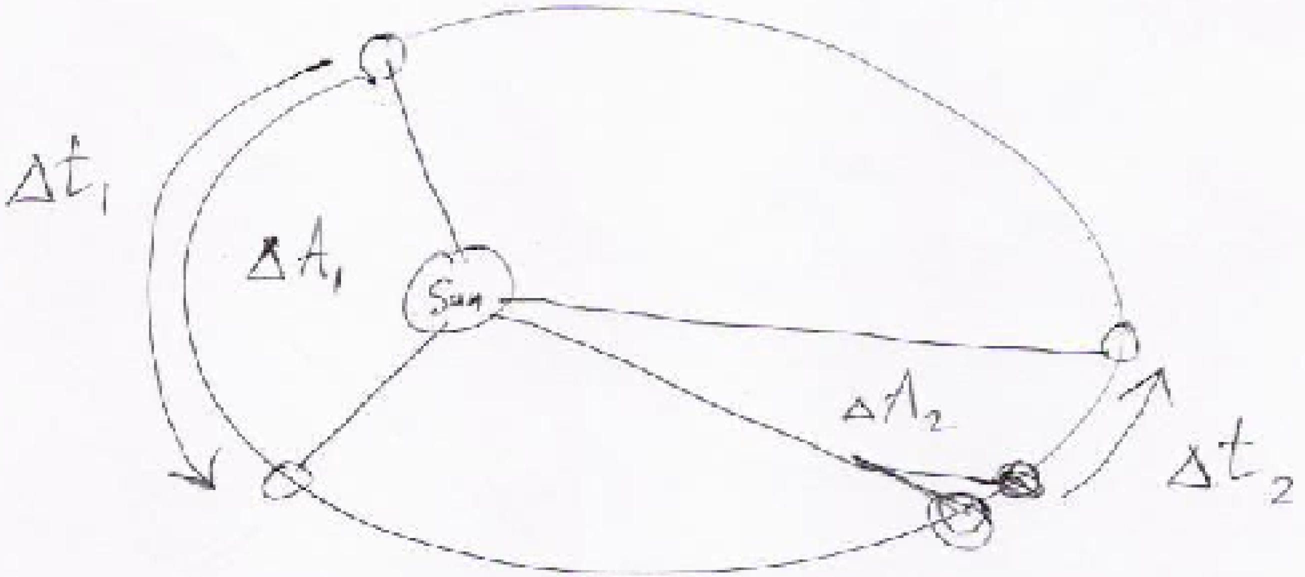
Kepler's Laws

1) Planets orbit the sun in ellipses with the sun at one focus.



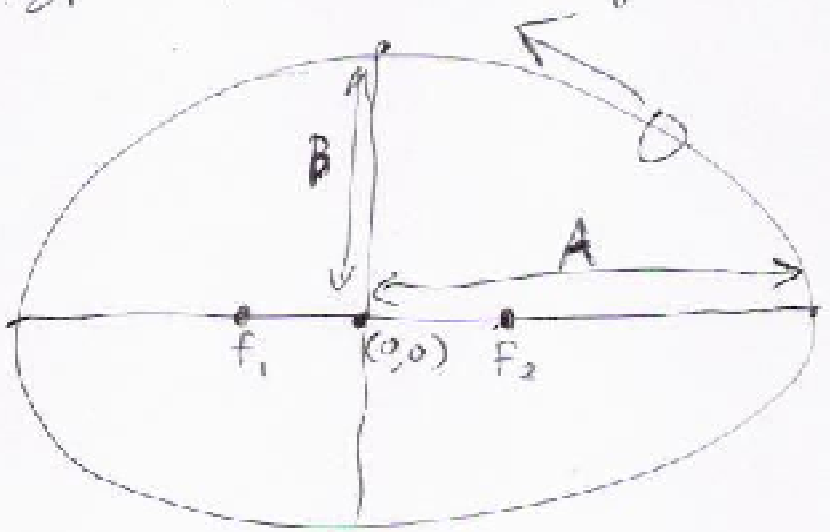
An ellipse is a closed curve s.t. pts on the curve have a constant sum of distances to two points called foci.

2) Planet "sweep out" sectors at a constant rate of area per time.



$$\frac{\Delta A_1}{\Delta t_1} = \frac{\Delta A_2}{\Delta t_2}$$

☆ 3) The period of a planet's orbit is proportional to the 3/2 power of its semimajor axis.



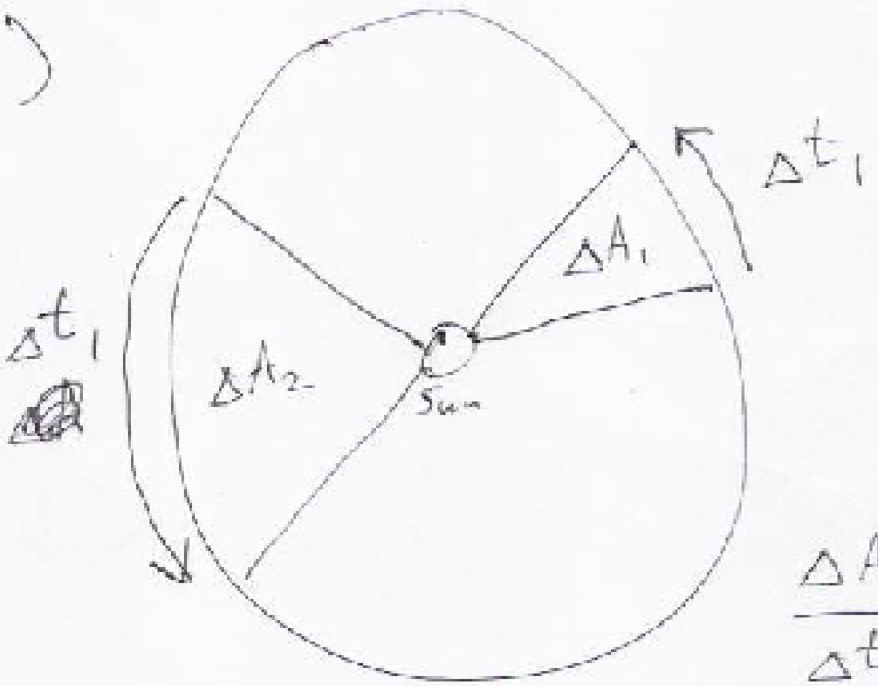
$$T \underbrace{\propto}_{} A^{3/2}$$

is proportional to.

Simplified Kepler's Laws:

1) The planets orbit in circles

2)



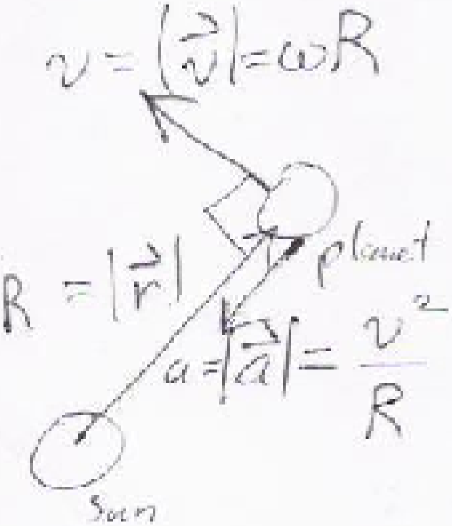
θ in radians

$$\Delta A_1 = \frac{1}{2} R^2 \Delta\theta_1$$

$$\Delta A_2 = \frac{1}{2} R^2 \Delta\theta_2$$

$$\frac{\Delta A_1}{\Delta t_1} = \frac{\Delta A_2}{\Delta t_2}$$

$$\frac{1}{2} R^2 \frac{\Delta\theta_1}{\Delta t_1} = \frac{1}{2} R^2 \frac{\Delta\theta_2}{\Delta t_2}$$



$$\omega = \frac{\Delta\theta_1}{\Delta t_1} = \frac{\Delta\theta_2}{\Delta t_2}$$

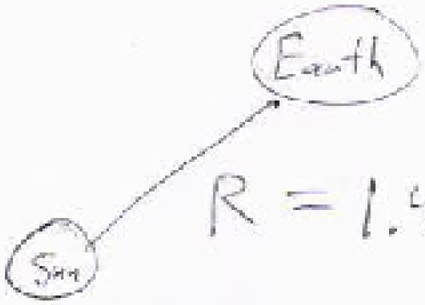
ω is constant.

$\Downarrow$

$\Leftarrow$ Uniform circular motion.

$v = \omega R$

$a = \frac{v^2}{R} = \omega^2 R$ $\qquad \omega = \frac{2\pi}{T}$ $\qquad T$ = period



$R = 1.496 \times 10^{11}\ \text{m}$

$T = 365.25\ \text{days} = 3.156 \times 10^{7}\ \text{s}$

$\omega = \dfrac{2\pi}{T} = 1.991 \times 10^{-7}\ \underbrace{\text{rad/s}}_{1/\text{s}}$

$\omega = \dfrac{360^\circ}{T} = 1.141 \times 10^{-5}\ \underbrace{\text{deg}}\text{/s}$

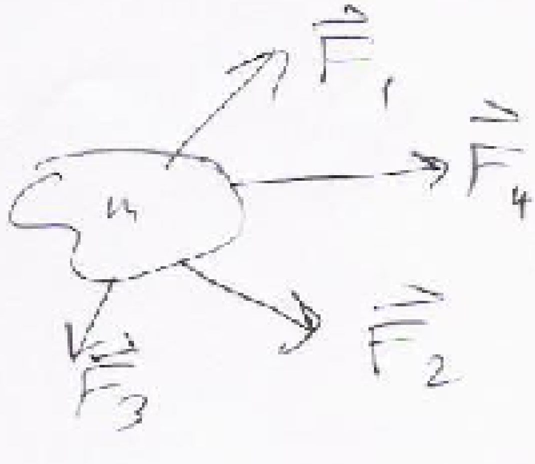
$\dfrac{2\pi}{360} = \dfrac{\pi}{180}$

$v = \omega R = 2.978 \times 10^{4}\ \text{m/s}$

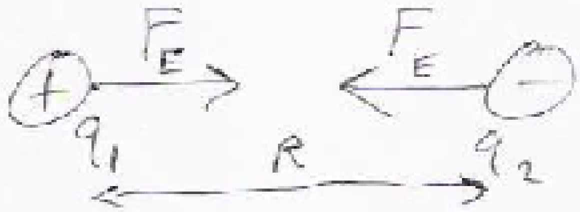
$a = \dfrac{v^2}{R} = \omega^2 R = 5.929 \times 10^{-3}\ \text{m/s}^2$

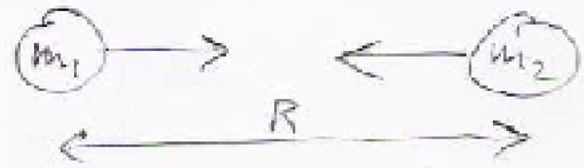
Newton

$\sum \vec{F} = \underbrace{m}_{\text{inertial mass}} \vec{a}$



Electrical force | Gravitational force





$$F_E = \frac{k|q_1 q_2|}{R^2} \qquad F_G = \frac{G m_1 m_2}{R^2}$$

k, G constants

q_1, q_2 electrical charges

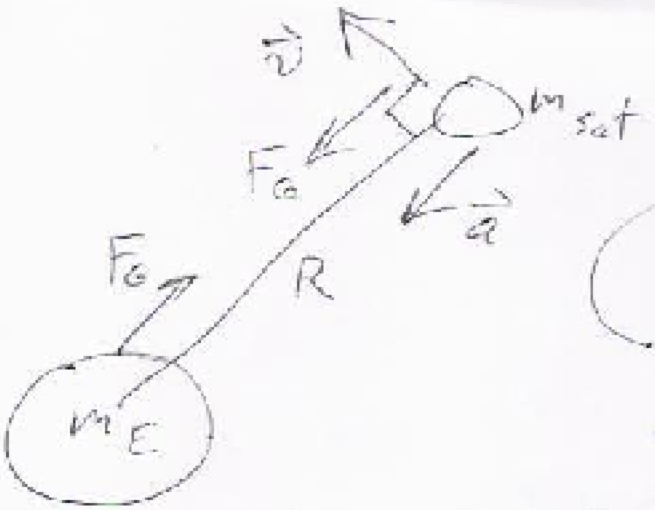
m_1, m_2 gravitational charges

inertial mass = gravitational charge

Gravity & curved spacetime coordinate systems are the same thing. - Einstein

(Galaxies curve path of light.)

At what radius from the earth's center would a satellite orbit in a circle with a period of 12 hours? What would its speed & acceleration be?



$$a = \frac{v^2}{R} = \omega^2 R$$

$$\omega = 2\pi / T$$

$$m_{sat}\, a = F_G = \frac{G m_E m_{sat}}{R^2}$$

$G = 6.67 \times 10^{-11}$
$N \cdot m^2/kg^2$

$$a = \frac{G m_E}{R^2}$$

$m_E = 5.97 \times 10^{24}$ kg

$$\omega^2 R = \frac{G m_E}{R^2}$$

$T = 12$ hours $= 43200$ s

$\Downarrow$

$\omega = 1.4544 \times 10^{-4} / s$

$$\omega^2 R^3 = G m_E$$

$$R^3 = \frac{G m_E}{\omega^2}$$

$$R = \sqrt[3]{\frac{G m_E}{\omega^2}} = 2.66 \times 10^7 \quad \underbrace{\sqrt[3]{\frac{N(m^2/kg^2)\cdot kg}{(1/s)^2}}}$$

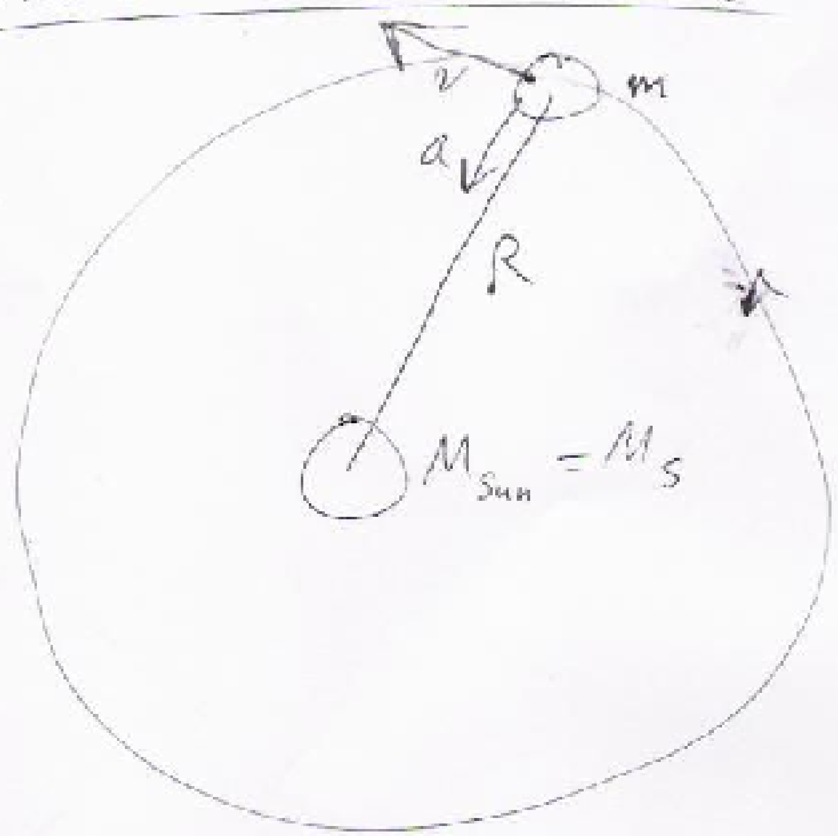
$$R = 2.66 \times 10^7 \ m$$

$$\underbrace{\sqrt[3]{\frac{(kg \cdot m/s^2)(m^2/kg^2)\cdot kg}{(1/s)^2}}}$$

$$\sqrt[3]{m^3} = m$$

altitude $= R - R_E$

Kepler's 3rd Law (for circles)



$$a = \frac{v^2}{R} = \omega^2 R$$

$$\frac{G M_S m}{R^2} = F_G = ma$$

$$\frac{G M_S}{R^2} = a = \omega^2 R$$

$$G M_S = \omega^2 R^3$$

$$\omega = \frac{2\pi}{T}$$

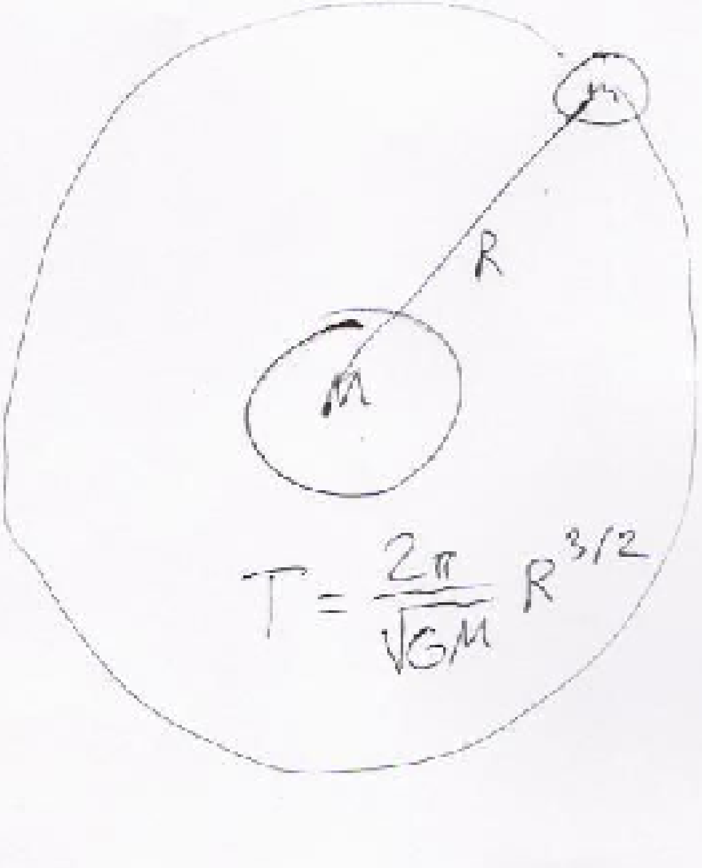
$$G M_S = \frac{4\pi^2}{T^2} R^3$$

$$T^2 G M_S = 4\pi^2 R^3$$

$$T^2 = \frac{4\pi^2 R^3}{G M_S}$$

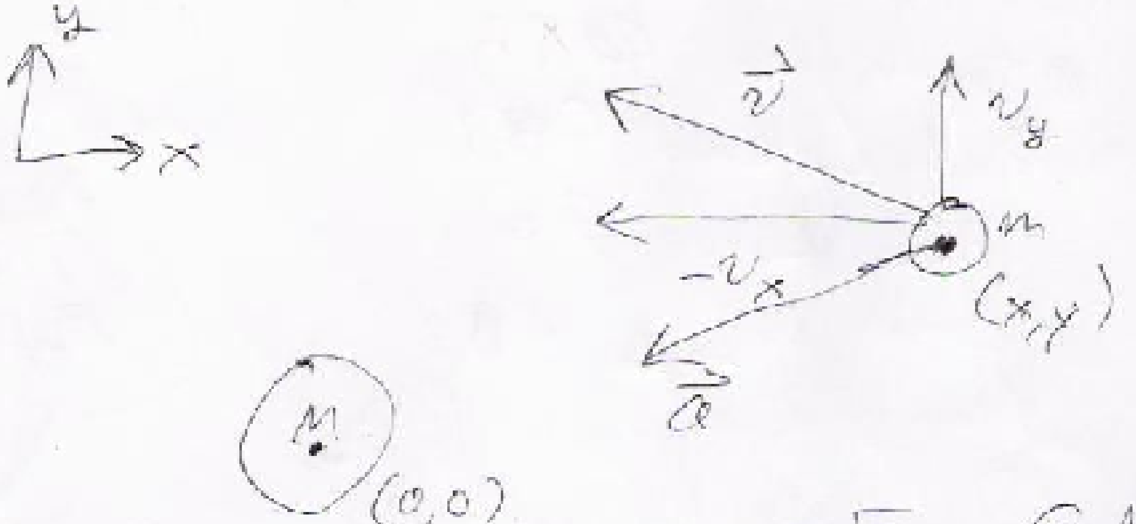
$$T = \frac{2\pi}{\sqrt{G M_S}} R^{3/2}$$

$$T \propto R^{3/2}$$

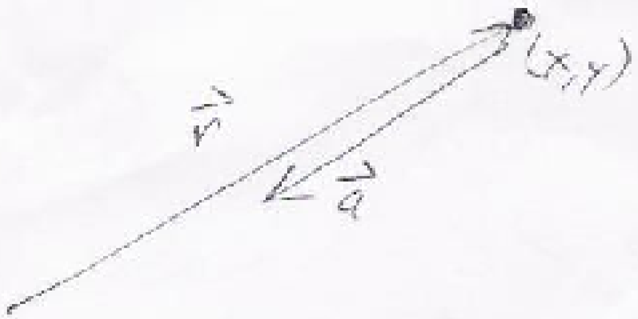


$$T = \frac{2\pi}{\sqrt{GM}} R^{3/2}$$

Simulating elliptical orbits:



$$ma = F_G = \frac{GMm}{R^2}$$



$$R^2 = x^2 + y^2$$

$$\hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{(x, y)}{\sqrt{x^2+y^2}} \qquad a = \frac{GM}{x^2+y^2}$$

$$\hat{a} = -\hat{r} = \frac{-(x, y)}{\sqrt{x^2+y^2}}$$

$$\vec{a} = |\vec{a}|\,\hat{a} = \frac{GM}{x^2+y^2} \cdot \frac{-(x, y)}{\sqrt{x^2+y^2}}$$

$$a_x = \frac{-GMx}{(x^2+y^2)^{3/2}} \qquad a_y = \frac{-GMy}{(x^2+y^2)^{3/2}}$$

$$dv_x = a_x \cdot dt \qquad dx = v_x \cdot dt$$

$$v_x = v_x + dv_x \qquad x = x + dx \qquad t = t + dt$$