Test on Thursday

Also: ≥ 45 exercises

Ch. 5: uniform circular motion ✓

6: universal gravitation ✓

7: work & kinetic energy ✓

8: potential energy ✓ & power ✓

9: momentum

Physics I is cumulative:

chapters 1 - 4 & 5 - 1 get used in later chapters.

Work done by varying force:

$$\frac{1}{2}m(v^2 - v_0^2) = \Delta K = W = \int_{x_0}^{x} \vec{F} \cdot d\vec{r}$$

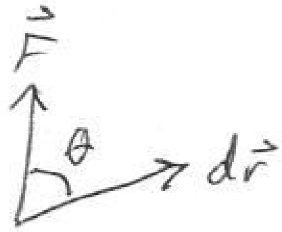
$$\vec{F} \cdot d\vec{r} = F\,dr \cos\theta$$

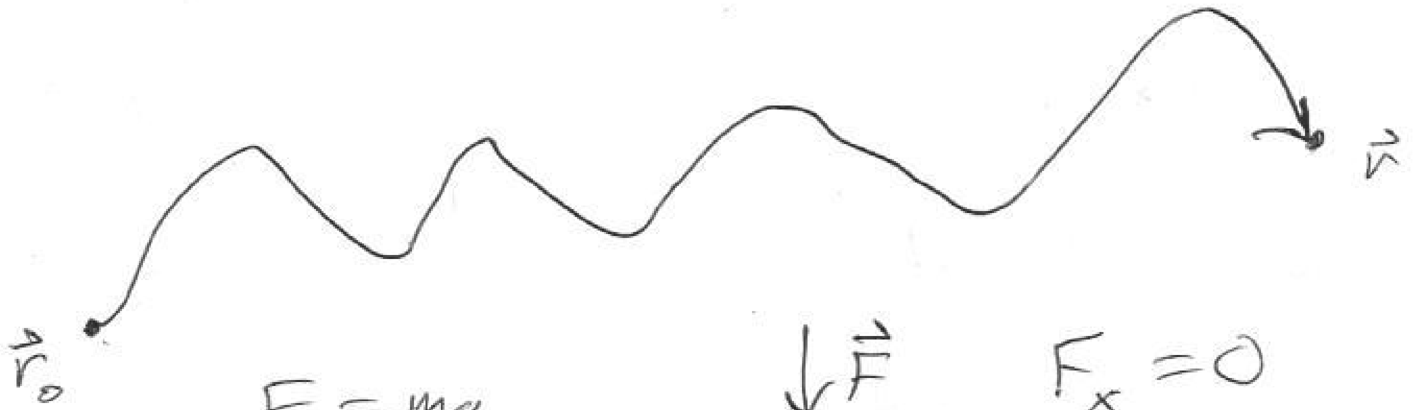
$$\vec{F} \cdot d\vec{r} = F_x\,dx + F_y\,dy + F_z\,dz$$

If $\vec{F}$ & direction of $d\vec{r}$ constant,

then $\quad W = F_{\parallel}(x - x_0) = F_{\parallel}\Delta x$

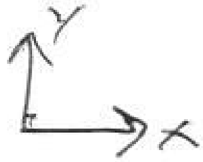
where $\quad F_{\parallel} = F\cos\theta$ where





$F = mg \qquad \downarrow \vec{F} \qquad F_x = 0$

$F_y = -mg$

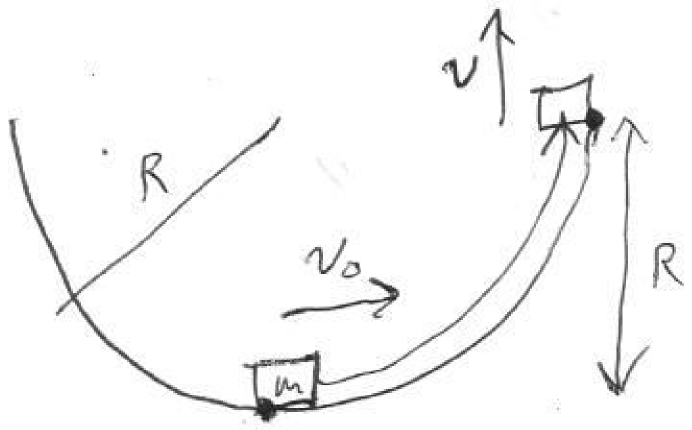


$$W = \int_{\vec{r}_0}^{\vec{r}} \vec{F} \cdot d\vec{r} = \int_{\vec{r}_0}^{\vec{r}} \left(F_x\,dx + F_y\,dy\right)$$

$$= \int_{x_0}^{x} F_x\,dx + \int_{y_0}^{y} F_y\,dy = \int_{y_0}^{y} -mg\,dy$$

$$= -mg\int_{y_0}^{y} dy = -mgy\Big|_{y_0}^{y} = -mg(y - y_0)$$

$v = ?$



semicircle

$R = 1.30\ m$

block mass

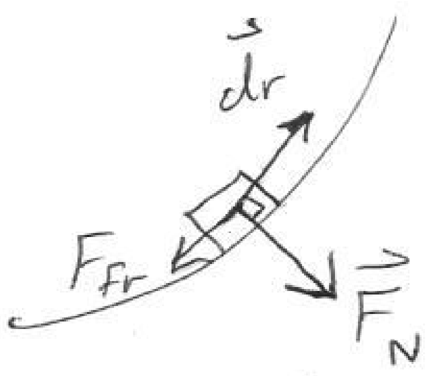
$m = 6.30\ kg$

$\mu_k = 0.41$

$v_0 = 15.62\ m/s$

$$W_g = -mg(y - y_0) = -mgR$$

$$\Delta K = \frac{1}{2}m(v^2 - v_0^2) = W_g + W_N + W_{fr}$$



$$d\vec{r} \perp \vec{F}_N \Rightarrow d\vec{r} \cdot \vec{F}_N = 0$$

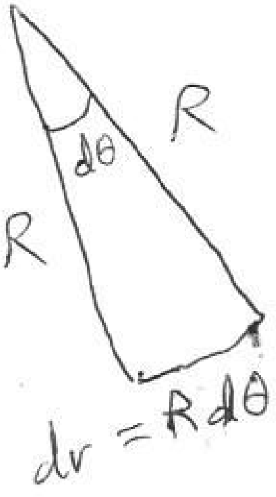
$$\cos 90^\circ = 0$$

$$W_N = 0$$

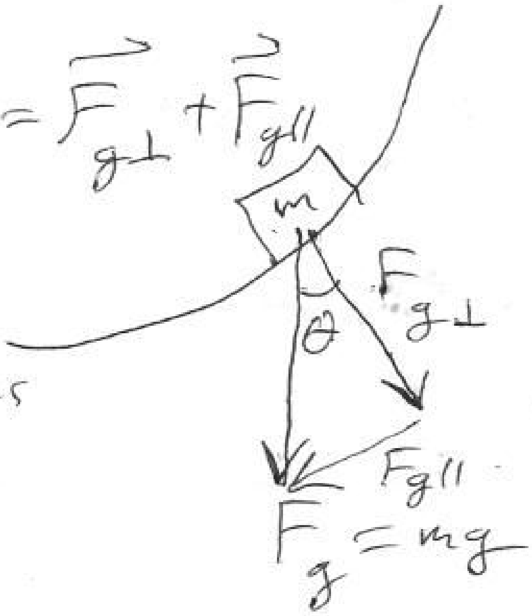
$$W_{Fr} = \int_{\vec{r}_0}^{\vec{r}} \vec{F}_{Fr} \cdot d\vec{r} = \int_{\vec{r}_0}^{\vec{r}} F_{fr}\, dr \underbrace{\cos 180^\circ}_{-1}$$

$$W_{fr} = -\int_{\theta_0}^{\theta} F_{fr} \underbrace{R\, d\theta}_{dr} \qquad F_{fr} = \mu_k F_N$$

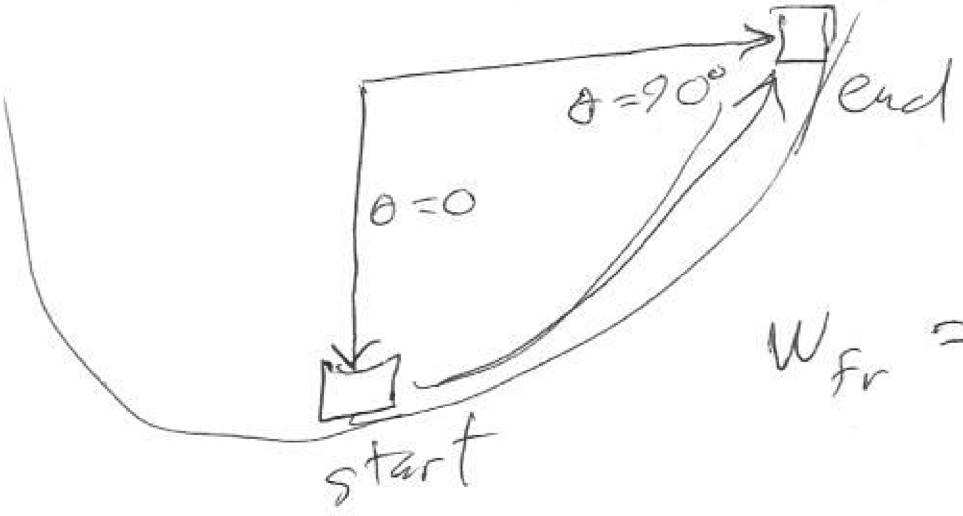
$$F_N = F_{g\perp} = mg\cos\theta$$



$$\vec{F}_g = \vec{F}_{g\perp} + \vec{F}_{g\parallel}$$



θ = angle with vertical of radius



$$W_{fr} = -\int_{\theta=0^\circ}^{\theta=90^\circ} \underbrace{\mu_k mg\cos\theta}_{F_{fr}} R\,d\theta$$

$$W_{fr} = -\int_0^{\pi/2} \underbrace{\mu_k mgR}_{\text{constant}} \cos\theta\,d\theta$$

$$W_{fr} = -\mu_k mgR \sin\theta \Big|_0^{\pi/2} = -\mu_k mgR(1-0)$$

$$W_{fr} = -\mu_k mgR$$

$$\frac{1}{2}m(v^2 - v_0^2) = -mgR + 0 - \mu_k mgR$$

$$\frac{1}{2}(v^2 - v_0^2) = -gR(1 + \mu_k)$$

$$v^2 - v_0^2 = -2gR(1 + \mu_k)$$

$$v^2 = v_0^2 - 2gR(1 + \mu_k)$$

$$v = \sqrt{v_0^2 - 2gR(1 + \mu_k)}$$

It's always an integral, but some integrals are easy: $\int_3^5 k\,dx = k(5-3) = 2k$

<u>if</u> k constant

$$\int_{x_0}^{x} k\,dx = k\Delta x = k(x-x_0)$$

$\vec{F} = d\vec{p}/dt$

Units: Force: $1\ \mathrm{N} = 1\ \mathrm{kg\cdot m/s^2}$

$(\vec{p} = m\vec{v})$ Momentum: $\mathrm{kg\cdot m/s}$

Work & Energy: $1\ \mathrm{J} = 1\ \mathrm{N\cdot m} = 1\ \mathrm{kg\cdot m^2/s^2}$

$W_F = \int \vec{F}\cdot d\vec{r}$

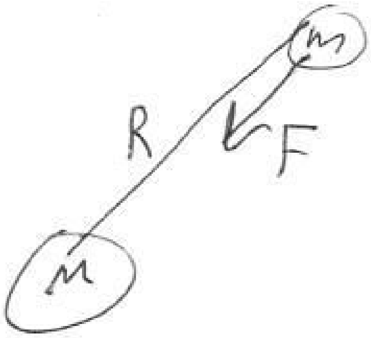
Power: $1\ \mathrm{W} = 1\ \mathrm{J/s} = 1\ \mathrm{kg\cdot m^2/s^3}$

$$P = dW/dt = \vec{F}\cdot\vec{v} = Fv\cos\theta$$

Potential energy:

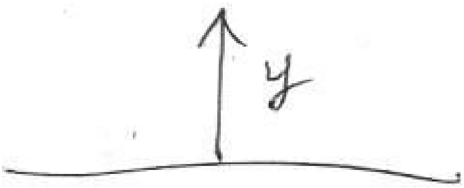
<u>If</u> work done by a force F is independent of path taken, then define $\Delta U_F = -W_F$.

Gravity: $$\Delta U_G = \left(-\frac{GMm}{R}\right) - \left(-\frac{GMm}{R_0}\right)$$



$$\Delta U_G = -GMm\,\Delta\left(\frac{1}{R}\right)$$

Near earth's surface:



$$g = \frac{GM_E}{R_E^2}$$

$$\Delta U_G \approx mg\Delta y$$

$$dU_G \approx mg\,dy$$

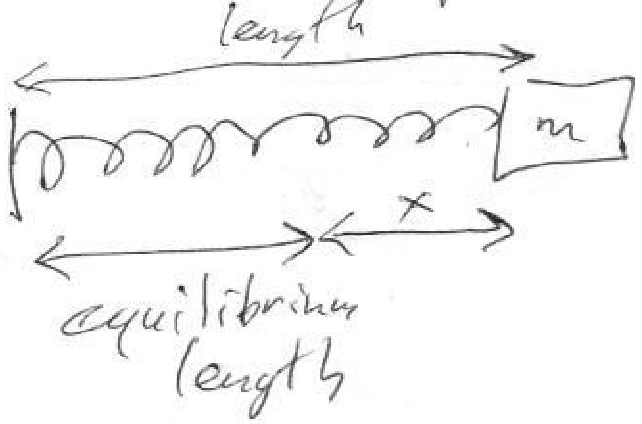
$$dU_G = d\left(-\frac{GM_E m}{R}\right) = \frac{GM_E m}{R^2}\,dR$$

If $R \approx R_E$, then $dU_G \approx \frac{\overbrace{GM_E}^{g} m}{R_E^2}\,dR$

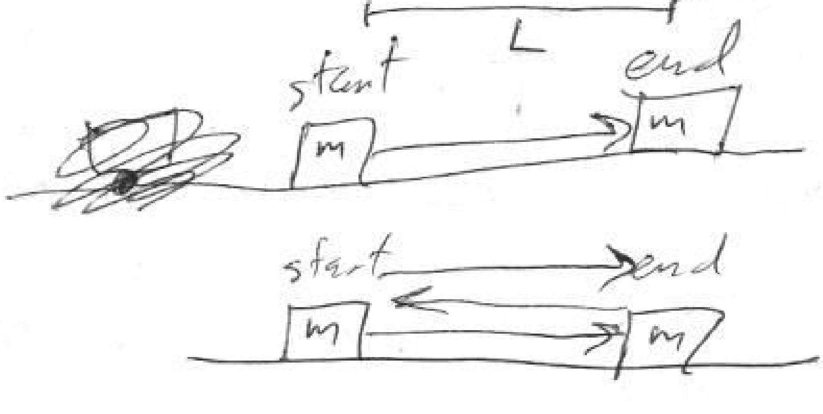
$$= gm\,dR = gm\,dy = mg\,dy$$

Springs: $\Delta U_{sp} = \frac{1}{2} k \,\Delta(x^2)$

k = spring constant



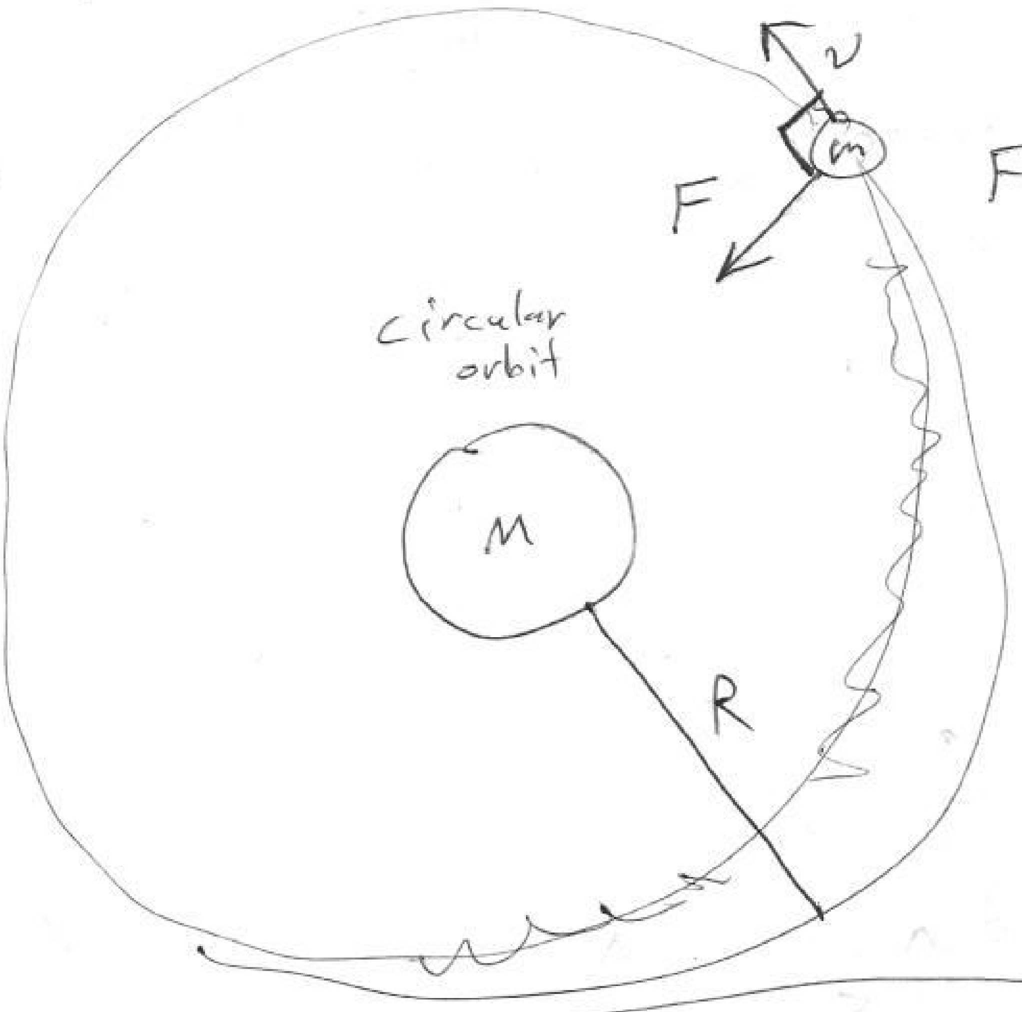
Friction's work depends on path taken.



$W_{fr} = -\mu_k mgL$

$W_{fr} = -3\mu_k mgL$

3 times work done.



$$F = \frac{GMm}{R^2}$$

$$\vec{F} \perp \vec{v}$$

$\Downarrow$

no work done

$\Downarrow$

kinetic energy constant

$\Downarrow$

v constant

$$G = 6.67 \cdot 10^{-11} \underbrace{N \cdot m^2/kg^2}_{m^3/(kg \cdot s^2)}$$

uniform circular motion

$$v = \frac{2\pi R}{T} \qquad a = \frac{v^2}{R}$$

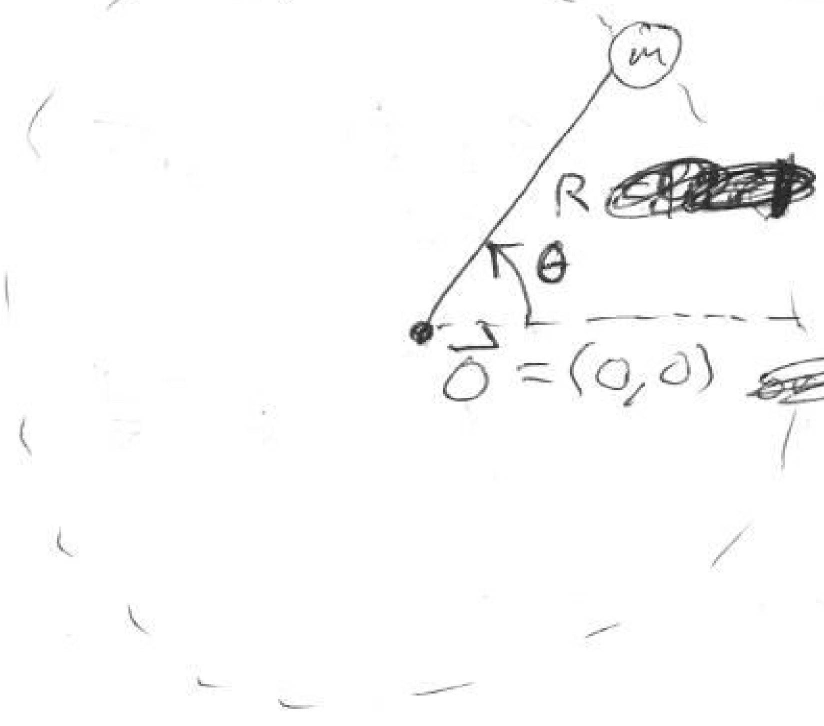
$$\frac{GMm}{R^2} = F = ma = \frac{mv^2}{R} = \frac{4\pi^2 R^2}{T^2} \cdot \frac{m}{R}$$

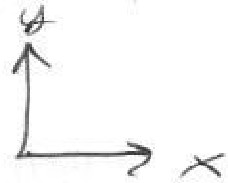
$$\frac{GM}{R^2} = \frac{4\pi^2 R}{T^2} \Rightarrow 4\pi^2 R^3 = GMT^2$$

$$T = \frac{2\pi}{\sqrt{GM}} R^{3/2}$$ ⇐ Kepler's 3rd Law for circular orbits

"v constant" is Kepler's 2nd Law for circular orbits.

Circular motion:





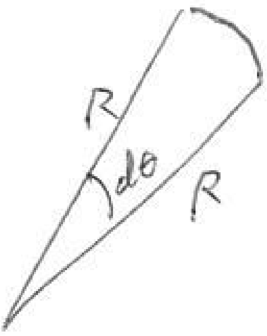
$$x = R\cos\theta$$
$$y = R\sin\theta$$

angular velocity $\omega = \frac{d\theta}{dt}$

angular acceleration $\alpha = \frac{d\omega}{dt}$

Uniform circular motion

means $\alpha = 0$, that is, ω constant.



$$|d\vec{r}| = R\,d\theta \Rightarrow v = |\vec{v}| = \left|\frac{d\vec{r}}{dt}\right| = \left|\frac{R\,d\theta}{dt}\right|$$

$$v = R|\omega|$$

$$\alpha = \frac{d\omega}{dt} = 0 \Rightarrow \omega \text{ constant} \Rightarrow \frac{d\theta}{dt} \text{ const}$$

$$\Downarrow$$

$$v = \frac{2\pi R}{T} \Leftarrow \frac{2\pi}{T} = \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt} = \omega$$

$$a = \frac{v^2}{R} = \omega^2 R \Rightarrow F = ma = \frac{mv^2}{R}$$