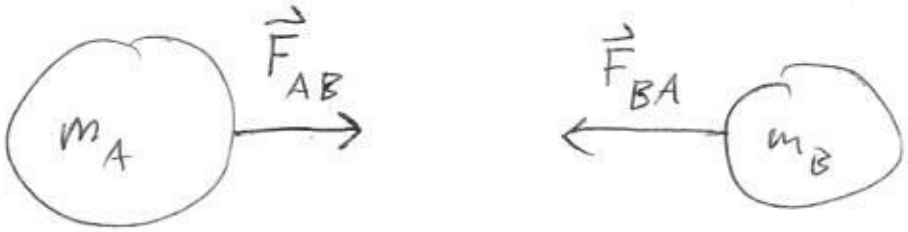


position $\vec{r}_A$ position $\vec{r}_B$

center of mass: $\vec{r}_{cm} = \dfrac{m_A \vec{r}_A + m_B \vec{r}_B}{m_A + m_B}$

$\vec{F}_{AB} = \dfrac{d\vec{p}_A}{dt} = m_A \vec{a}_A$ & $\vec{F}_{BA} = \dfrac{d\vec{p}_B}{dt} = m_B \vec{a}_B$

Newton's 3rd Law $\vec{F}_{BA} = -\vec{F}_{AB}$

$$\vec{a}_{cm} = \frac{m_A \vec{a}_A + m_B \vec{a}_B}{m_A + m_B} = \frac{\vec{F}_{AB} + \vec{F}_{BA}}{m_A + m_B} = 0$$

$\vec{v}_{cm}$ constant

Look at collisions in the C.M. reference frame.

Collision from last time:

$m_A = 5.0\ kg$ $m_B = 6.0\ kg$

$v_{Ax} = 4.0\ m/s$ $v_{Bx} = -1.0\ m/s$

$$\frac{d\vec{r}_{cm}}{dt} = \vec{v}_{cm} = \frac{m_A \vec{v}_A + m_B \vec{v}_B}{m_A + m_B} = \frac{14}{11}\ m/s$$

in the positive x direction

Same before & after collision.

$\vec{u}_A$ = velocity of mass $m_A$ relative to the c.m.

$\vec{u}_A = \vec{v}_A - \vec{v}_{cm}$ $\quad u_{Ax} = \left(4.0 - \frac{14}{11}\right) m/s = \frac{30}{11}\ m/s$

$\vec{u}_B$ = ——— $m_B$ ———

$\vec{u}_B = \vec{v}_B - \vec{v}_{cm}$ $\quad u_{Bx} = \left(-1.0 - \frac{14}{11}\right) m/s = -\frac{25}{11}\ \frac{m}{s}$

Before collision.

After completely inelastic collision:

(stick together) $\vec{u}_A^{\,*} = \vec{u}_B^{\,*} = \vec{0} \Rightarrow u_{Ax} = u_{Bx} = 0$

After completely elastic collision:


& $\vec{u}_B^{\,*} = -\vec{u}_B$.

(bounce off at same speeds)

$\vec{u}_A^{\,*} = -\vec{u}_A$

$u_{Ax} = -\frac{30}{11}\ m/s$

$u_{Bx} = \frac{25}{11}\ m/s$

Go back to original reference frame.

$$\vec{V}_A = \vec{u}_A^{\,*} + \vec{V}_{cm} = \begin{cases} \left(0 + \frac{14}{11}\right)\frac{m}{s} = \frac{14}{11}\frac{m}{s} & \text{comp. inela.} \\ \left(-\frac{30}{11} + \frac{14}{11}\right)\frac{m}{s} = -\frac{16}{11}\frac{m}{s} & \text{comp. elastic} \end{cases}$$

(in x direction)

$$\vec{V}_B = \vec{u}_B^{\,*} + \vec{V}_{cm} = \begin{cases} \left(0 + \frac{14}{11}\right)\frac{m}{s} = \frac{14}{11}\frac{m}{s} & \text{comp. inelastic.} \\ \left(\frac{25}{11} + \frac{14}{11}\right)\frac{m}{s} = \frac{39}{11}\frac{m}{s} & \text{comp elastic} \end{cases}$$

After collission, in original reference frame.