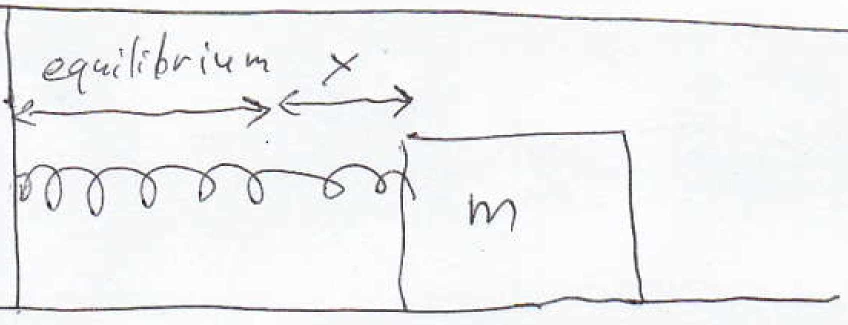
Today (3/20) } → Ch. 14. (Oscillations)
(3/22) } → Simulations & exercises

3/27: Last day for approval of your simulation choice

4/19: Simulations due



k = spring constant

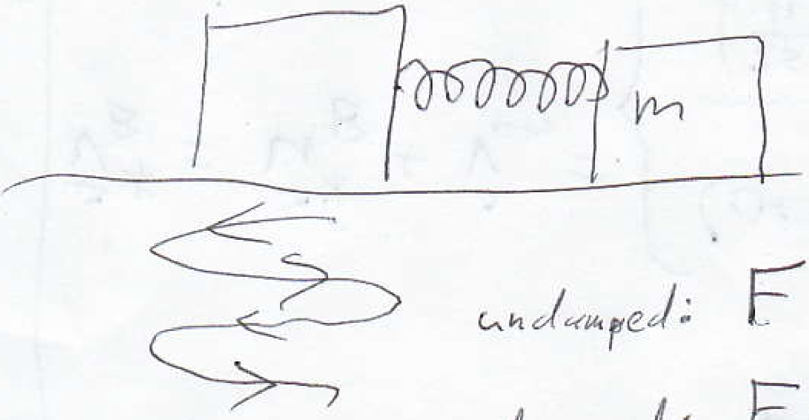
↓

undamped oscillation: $F = -kx$

damped oscillation: $F = -kx - bv$

↑ from friction inside spring

driven oscillation:



undamped: $F = -kx + \underbrace{F_0 \cos(\omega_0 t + \phi_0)}$

damped: $F = -kx - bv + F_0 \cos(\omega_0 t + \phi_0)$

Resonance: ω_0 close to $\omega = \sqrt{\frac{k}{m}}$

$\Downarrow$

bigger oscillations

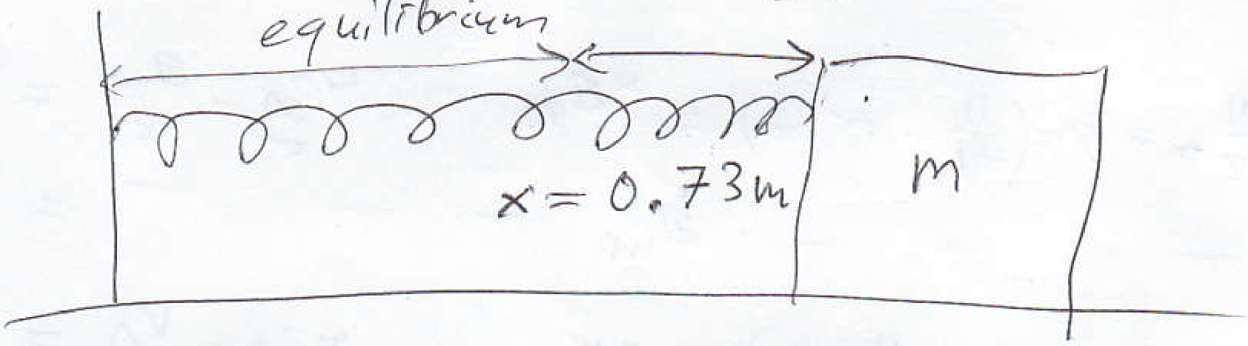
Simulating $F = -kx$

$$ma = F = -kx$$

$$-\frac{kx}{m} = a = \frac{dv}{dt} \qquad v = \frac{dx}{dt}$$

To simulate, pick starting time t, starting position x, starting velocity v; pick mass m; spring constant k.

start $t=0$



$m = 5.8\ kg \qquad v = 0$

$k = 15\ N/m$

Pick small time increment dt.
$= 0.1s$

$$-\frac{kx}{m} = a = \frac{dv}{dt} \qquad v = \frac{dx}{dt}$$

Program: $a = -k*x/m$

$dv = a*dt$

$v = v + dv$

$dx = v*dt$

$x = x + dx$

$t = t + dt$

Repeat

Mathematical analysis:

$$-\frac{kx}{m} = a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

amplitude

$$\frac{d^2}{dt^2}\left(A\cos\left(\overbrace{\omega t}^{\text{rad}} + \overbrace{\phi}^{\text{rad}}\right)\right)$$

measured in meters

$A > 0$

angular frequency

rad/s

phase shift tells you where you

ϕ, φ, Φ

$$\frac{d}{dt}\left(\frac{d}{dt}\left(A\cos(\omega t+\phi)\right)\right)$$

$$= A\frac{d}{dt}\left(\frac{d}{dt}\cos(\omega t+\phi)\right)$$

$$= A\frac{d}{dt}\left(-\sin(\omega t+\phi)\cdot\underbrace{\frac{d}{dt}(\omega t+\phi)}_{\omega}\right)$$

$$= -A\omega\frac{d}{dt}\sin(\omega t+\phi)$$

$$= -A\omega\cos(\omega t+\phi)\underbrace{\frac{d}{dt}(\omega t+\phi)}_{\omega}$$

$$= -A\omega^2\cos(\omega t+\phi)$$

$$= -\omega^2\left(A\cos(\omega t+\phi)\right)$$

$$x = A\cos(\omega t+\phi)$$

$$\Downarrow$$

$$a = d^2x/dt^2 = -\omega^2 x$$

Compare to: $a = -kx/m$

$$\omega = \sqrt{\frac{k}{m}}$$

1 cycle = 2π radians

angular frequency ω = # radians per second

frequency $f = \frac{\omega}{2\pi}$ = # cycles $\underbrace{\text{per second}}_{\text{Hertz}}$

Hz

(Other units for f & ω are possible, like rpm (revolutions per minute).)

period $T = \frac{1}{f} = \frac{2\pi}{\omega} = \underbrace{\text{time/cycle}}_{\text{seconds}}$

(Other units, like minutes, hours, etc. are possible for T.)

$$T = \frac{2\pi}{\sqrt{k/m}} = 2\pi\sqrt{m/k}$$

$k = 150\ N/m \qquad m = 5.8\ kg$

Simulation: $T = 1.23\,s$

Theory: $2\pi\sqrt{m/k} = 2\pi\sqrt{\frac{5.8\,kg}{150(kg \cdot m/s^2)/m}}$

$$= 2\pi\sqrt{\frac{5.8}{150}s^2} = 2\pi\sqrt{\frac{5.8}{150}}\,s = \underbrace{1.2}_{\text{2 sig figs}}355\ldots s$$

$ma = F = -kx - bv$ damped oscillation

$$\frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt}$$

$$x = \underbrace{e^{-\gamma t} A_0}\cos(\omega' t + \phi)$$

A = amplitude exponentially decays

γ & ω' are solved for...

involves quadratic equation

$$\gamma = \frac{b}{2m} \quad \& \quad \omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$