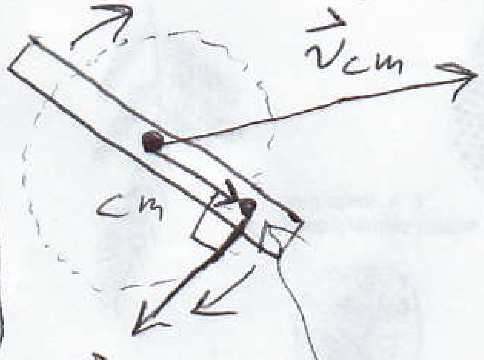
motion = translational motion + rotational motion

rotating pen in freefall

velocity of c.m. relative to ground



$$M\frac{d\vec{v}_{cm}}{dt} = \underbrace{\sum \vec{F}_{ext}}_{= Mg}$$

Mg ← Free fall

c.m. is center of mass:

From Ch. 9

$$\sum_{\text{pieces}} \vec{x}_{piece}\, m_{piece}/M$$

$M = \sum_{\text{pieces}} m_{piece}$

M = total mass

velocity relative to c.m.

the velocity $\vec{v}$ (relative to ground) of that point is

$$\vec{v} = \vec{v}_{cm} + \vec{v}_{rot}$$

Energy of pen: $K + U = E$

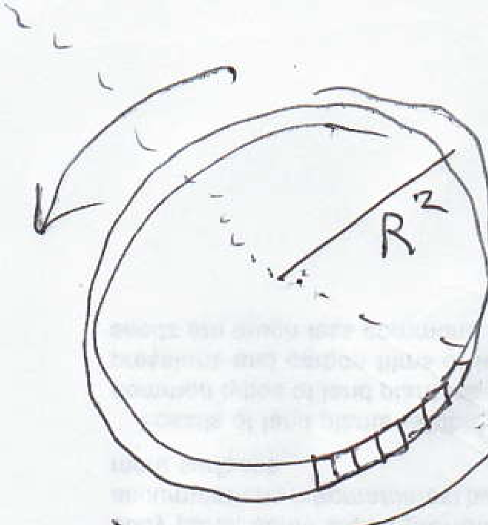
$U = Mgh$ (M: mass, h: height of c.m.)

$$K = \underbrace{\frac{1}{2}Mv_{cm}^2}_{\text{translational K.E.}} + \underbrace{\frac{1}{2}I\omega^2}_{\text{rotational kinetic energy}}$$

$\omega = \frac{2\pi}{T}$

T = period of rotation

I = moment of inertia.
= "rotational mass"



thin ring

$$I = MR^2$$

Case $v_{cm} = 0$

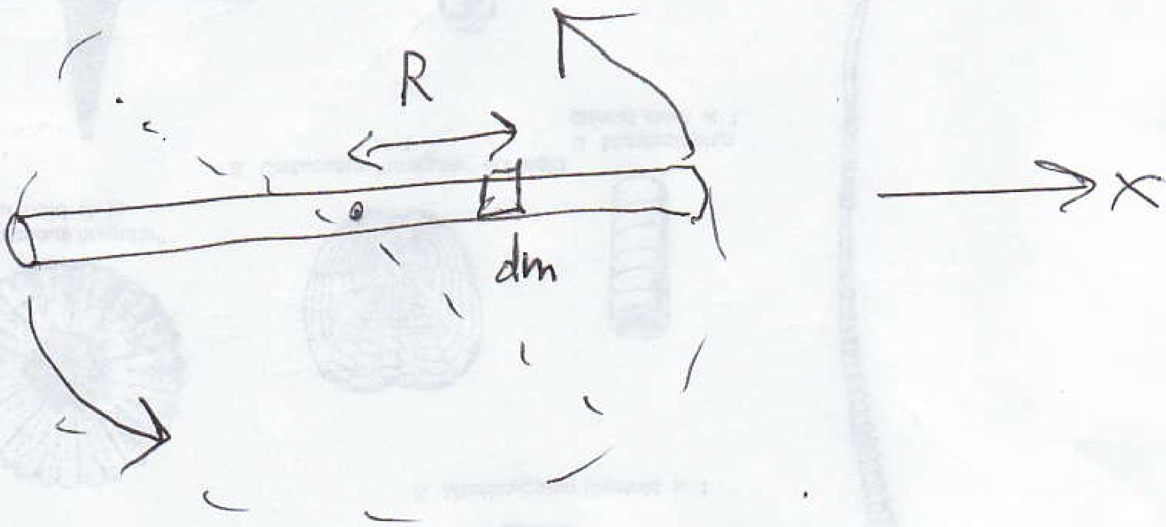
$$K_{ring} = \sum_{pieces} m_{piece} v^2 = \left(\sum_{pieces} m_{piece}\right)\omega^2 (R^2)$$

constant $v \Rightarrow v = \frac{2\pi R}{T} = \left(\frac{2\pi}{T}\right)R = \omega R$

$$I = \sum_{pieces} m_{piece} R^2 = \underbrace{M}_{\text{total mass}} R^2$$

$$K = I\omega^2$$

Moment of inertia of thin rod;
rotating about axis through center & perpendicular to rod:

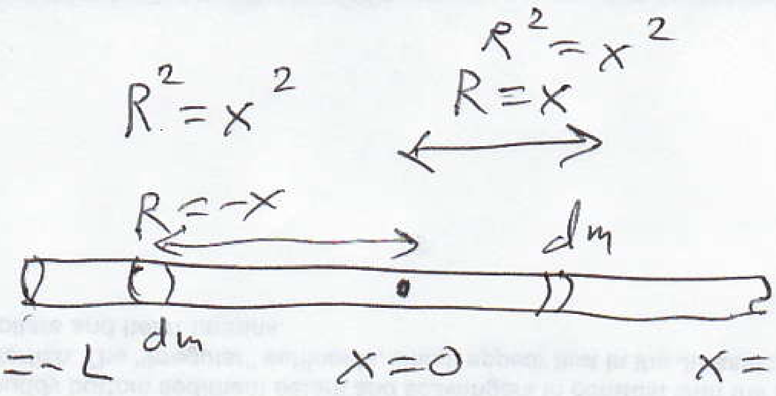


$$I = \sum_{\text{pieces}} m_{\text{piece}} R^2_{\text{piece}}$$ too many pieces!

$$I = \int_{\text{rod}} R^2 \, dm$$

For simplicity, let's assume uniform density. For a thin rod, that means

$$\frac{dm}{dx} = \frac{M}{2L}$$ where $2L$ length of rod, M mass of whole rod.



length of rod is $2L$

$$I = \int_{rod} R^2\, dm$$

$$I = \int_{x=-L}^{L} x^2 \frac{dm}{dx}\, dx$$

If uniform density holds, then the center of mass is the same as the geometric center.

$$I = \int_{-L}^{L} x^2 \frac{M}{2L}\, dx = \frac{M}{2L}\int_{-L}^{L} x^2\, dx$$

$$= \left(\frac{M}{2L}\frac{x^3}{3}\right)\Bigg|_{-L}^{L} = \frac{M}{2L}\left(\frac{L^3}{3} - \frac{(-L)^3}{3}\right)$$

$$= \frac{M}{6L}\left(L^3 - (-L^3)\right) = \frac{M}{6L}\left(2L^3\right) = \frac{L^2 M}{3}$$

If l = length of rod, then $L = l/2$

$$I = \frac{(l/2)^2 M}{3} = \frac{l^2 M/4}{3} = \frac{1}{12} l^2 M$$

See p. 260.

For computing I, see table on page 260 & parallel axis theorem (p. 264) for shortcuts

Translational

$\vec{v} = d\vec{x}/dt$

$\vec{a} = d\vec{v}/dt$

$\sum \vec{F} = M\vec{a}$

assumes M is constant

$\sum \vec{F} = \dfrac{d\vec{p}}{dt}$

always true

$\vec{p} = M\vec{v}$

momentum

Rotational

$\vec{\omega} = d\vec{\theta}/dt$

$\vec{\alpha} = d\vec{\omega}/dt$

$\sum \vec{\tau} = I\vec{\alpha}$

assuming rotating about ~~fixed~~ axis of symmetry.

$\sum \vec{\tau}_{cm} = \dfrac{d\vec{L}_{cm}}{dt}$

always true

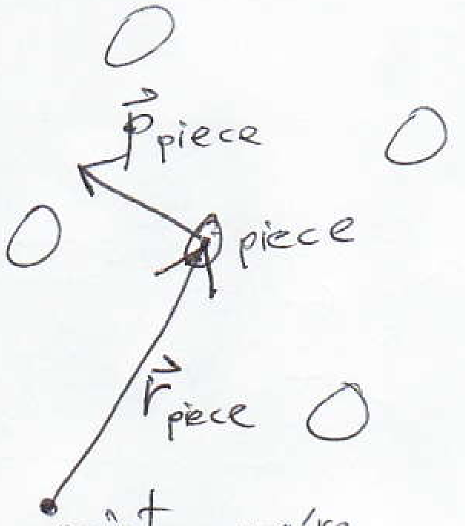
$|\vec{L}_{\cancel{\text{point}}\ \text{ring (thin)}}| = MRv = Rp$

thin ring

$\vec{L} = \sum_{\text{pieces}} \vec{r}_{\text{piece}} \times \vec{p}_{\text{piece}}$

angular momentum

direction of $\vec{L}$ is that of the half of the axis of rotation about which rotation is counterclockwise



point we're measuring rotation relative to.

$\vec{r} \times \vec{p} = -\vec{p} \times \vec{r}$ (See 11-2)

Anyone want to simulate a spinning top? (See 11-7)

$$\frac{d\vec{L}}{dt} = \sum_{pieces} \left(\frac{d\vec{r}_{piece}}{dt} \times \vec{p}_{piece} + \vec{r}_{piece} \times \frac{d\vec{p}_{piece}}{dt} \right)$$

$$= \sum_{pieces} \vec{\tau} = \sum_{pieces} \vec{r}_{piece} \times \vec{F}_{piece}$$

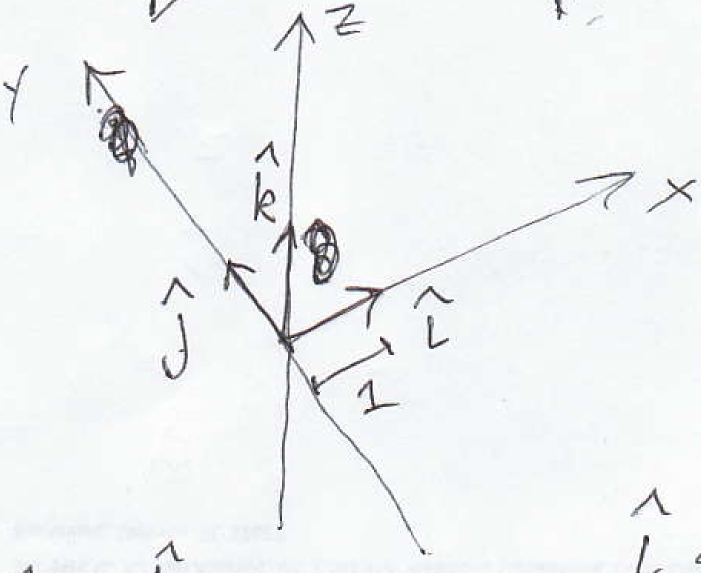
(See 11-4) if $\vec{r}$ measured relative to c-m.

$$|\vec{\tau}_{piece}| = |\vec{r}_{piece}| |\vec{F}_{piece}| \sin\phi$$

$$|\vec{A} \times \vec{B}| = |\vec{A}| |\vec{B}| \sin\phi$$

$$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}|\cos\phi$$

$$|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}|\sin\phi$$



$$(3,4,5) = 3\hat{i} + 4\hat{j} + 5\hat{k}$$

$$\hat{i} \times \hat{j} = \hat{k}$$
$$\hat{j} \times \hat{i} = -\hat{k}$$

$$\hat{i} \times \hat{i} = \vec{0}$$
$$\hat{j} \times \hat{j} = \vec{0}$$
$$\hat{k} \times \hat{k} = \vec{0}$$

$$\hat{k} \times \hat{i} = \hat{j}$$
$$\hat{i} \times \hat{k} = -\hat{j}$$

$$\hat{i} \times \hat{j} = \hat{k}$$
$$\hat{j} \times \hat{i} = -\hat{k}$$

$$\hat{j} \times \hat{k} = \hat{i}$$
$$\hat{k} \times \hat{j} = -\hat{i}$$

$\vec{A} \times \vec{B}$ is always perpendicular to $\vec{A}$ & perpendicular to $\vec{B}$.

torque: $\vec{\tau} = \vec{r} \times \vec{F}$ for each piece

$$|\vec{\tau}|_{piece} = |\vec{r}_{piece}|\,|\vec{F}_{piece}| \sin\phi$$